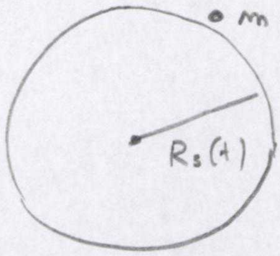
Petnica

(1) Friedmann equations + acceleration of the Universe

Newtonian cosmology:



$$F = -\frac{G M_s m}{R_s^2}$$

$$\frac{d^2 R_s}{dt^2} = -\frac{G M_s}{R_s^2(t)} \longrightarrow \frac{1}{2}\left(\frac{dR_s}{dt}\right)^2 - \frac{G M_s}{R_s(t)} = U = \text{const}$$

$$M_s = \frac{4\pi}{3}\rho(t) R_s(t)^3$$

- The assumption of homogeneous density is self-consistent
- No acceleration inside a hollow sphere: it applies to a homogeneous Universe

$R_s(t) = a(t)\, r_s$ $a(t)$ scale-factor

$$\boxed{\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) + \underbrace{\frac{2U}{r_s^2}\frac{1}{a(t)^2}}}$$

Friedmann equation (Newtonian form)

Like a stone thrown from the Earth

> 0 expanding forever

< 0 bouncing and collapsing

$= 0$ $\dot a \to 0$

- Expansion of the Universe is not only a GR concept (within Hubble radius expansion is a matter of interpretation)

GR modification

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{k}{R_0^2}\frac{1}{a^2}$$

1) ρ is not the mass density but the energy density

Q $\quad E = \sqrt{m^2 + \vec{p}^{\,2}}$

$c = 1$

For example photons contribute to the energy density although $m = 0$

2) In GR the constant of integration acquires a geometrical meaning: it is the curvature of space (not space-time!)

$$\frac{1}{\text{time}^2} = \frac{1}{\text{length}^2}c^2$$

Instead of flat Euclidean space, a surface of a 3-sphere (think the 2d counterpart) or a negatively curved space

- Curvature can be observed not only through F. equations but also by its direct geometric effect

Gravity $\longleftrightarrow$ Geometry

Very small experimentally $\lesssim 1\%$ in FE

Why? We are going to come back to this curvature problem

$\frac{\dot{a}}{a} \equiv H(t)$

Hubble parameter

H_0 constant Hubble now, $H(t_0)$

Critical density

If you assume zero curvature

$$H(t)^2 = \frac{8\pi G}{3}\rho(t)$$

Measuring H_0 (we will see how) $\longrightarrow$ ρ_c critical density

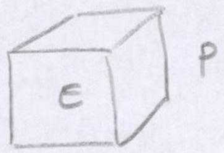
$$\rho_c^0 = \frac{3}{8\pi G} H_0^2$$

(5 hydrogen atoms / m^3 !)

$\sim 9 \cdot 10^{-27}$ kg/m^3 Very low by terrestrial standards

- For each component: $\Omega(t) = \frac{\rho(t)}{\rho_c(t)}$ They should sum to 1 if the Universe is flat

- We need an equation for the evolution of $\rho(t)$ to close eq.



The variation of the energy in the volume is the work done by pressure

$$\dot E = -p\dot V$$

$$\left(\frac{E}{V}\right)^{\cdot} = \frac{\dot E}{V} - \frac{E}{V}\frac{\dot V}{V} = \frac{\dot V}{V}(-p-\rho) \qquad V \propto a^3 \quad \frac{\dot V}{V} = 3\frac{\dot a}{a}$$

$$\boxed{\dot\rho = -3H(\rho + p)}$$

E.g. Matter $\rho \propto a^{-3}$

Radiation: large pressure $p = \frac{1}{3}\rho$ $\quad \rho \propto a^{-4}$ Photons lose energy

$$\dot a^2 = \frac{8\pi G}{3}\rho a^2 + \text{const}$$

$$2\dot a \ddot a = \frac{8\pi G}{3}\left(2\rho a \dot a + \dot\rho a^2\right) = \frac{8\pi G}{3} a^2 \left(2\rho H - 3H(\rho + p)\right)$$

$$= \frac{8\pi G}{3} a^2 (-\rho - 3p) H$$

Acceleration equation:

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)}$$

- Normal substances with positive pressure gives deceleration
- Deceleration = Attractive force, gravity
- For an ideal gas $PV = k_B N T$ Non-relativistic

$$P = \frac{N}{V} k_B T = \frac{\rho}{m} k_B T$$

For $k_B T \ll m$ (e.g. nitrogen molecules in air $\sim 10^{-12}$)

$\sim V^2$

For non-relativistic objects the pressure is negligible

($\rho = \frac{\text{mass}}{\text{volume}}$, $p = \frac{\text{mass} \cdot \ell}{t^2 \ell^2}$ $[\rho] = [P/c^2]$

$c \to +\infty$ Non-relativistic limit

Q)

Hubble law and luminosity distance

- Classic Hubble: $v(t) = \frac{dr}{dt} = \frac{d(a r_c)}{dt} = \frac{\dot{a}}{a} a r_c = H_0 r$

Velocity is measured with Doppler shift of emission / absorbtion lines.
Distance is measured assuming I know the luminosity

$$Z \equiv \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \qquad Z = \frac{v}{c} \quad \text{non-relativistic}$$

$H_0 \simeq 70$ km/s/Mpc

$$\boxed{Z = \frac{1}{c} H_0 r}$$

Hubble-law. Expansion of the Universe

This is just an approximation for nearby objects. For away objects send light that travels for a portion of the history of the Universe

Redshift $\leftrightarrow$ Time

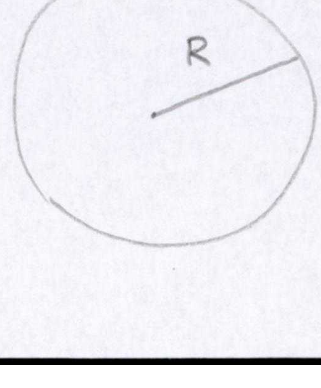
$$1 + Z = \frac{\lambda_{obs}}{\lambda_{em}} = \frac{a_{obs}}{a_{em}} \qquad \left(Z = \frac{a_{obs}}{a_{em}} - 1 \simeq H \Delta t = v \right)$$

It would correspond to faster than light

- Luminosity distance: $d_L \equiv \left(\frac{L}{4\pi f} \right)^{1/2}$ Observable quantity

Since $a(t)$ evolves as light propagates the distance is well defined only for nearby objects

$a(0) = 1$



$$d_L = (1+Z)R$$

$1+z$ since photon redshifts $\omega \propto \frac{1}{a}$

$1+z$ since the rate of photons (distance) increases

But how do I relate R with z?

$$\eta_{\mu\nu} = -\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$

$$\rightsquigarrow \quad -\Delta t^2 + a^2(t)\left(\Delta x^2 + \Delta y^2 + \Delta z^2\right)$$

Photons: $\Delta t = a(t)\,\Delta x$

$$X = \int_{t_{em}}^{t_0} \frac{dt}{a(t)} = \int_{a_{em}}^{a(0)=1} \frac{da}{a^2 H(a)} = \qquad a = \frac{1}{1+z}$$

$$= \int_0^z \frac{dz'}{H(z')}$$

Sensitive to all the history until emission

Discovery of acceleration!

$$d_L = (1+z)\int_0^z \frac{dz'}{H(z')} \qquad \left(\simeq H_0^{-1} z\right)$$

$$H(z) = H_0\left((1+z)^3\,\Omega_m + \Omega_\Lambda\right)$$

cosmological constant: $p = -\rho$

$\rho \propto a^0 = \text{const}$

Dimmer supernovae $\qquad \rho + 3p < 0$ acceleration

Cosmological constant

It corresponds to adding a constant to Friedmann equation

$$H^2(t) = \frac{8\pi G}{3}\left(\rho(t) + \rho_\Lambda\right)$$ (Introduced by Einstein to make the Universe static)

- ρ_Λ is compatible with all the symmetries of GR
- in QM actually we expect this term to be there

For example. Take EM field. Each EM wave with wavevector k is an independent harmonic oscillator

$$\rho_\Lambda \supset \int_0^{\Lambda_{UV}} \frac{d^3k}{(2\pi)^3} \underbrace{\frac{1}{2}k}_{\frac{\hbar\omega}{2} \text{ is the zero energy of h.o.}} \sim \Lambda_{UV}^4$$

We do not really know how to calculate (interactions?) but the value observed in the Universe is $\rho_\Lambda \sim (10^{-3}\,eV)^4$

We know particle physics and QED at TeV scales and maybe up to $M_{Pl} \sim 10^{19}$ GeV scale

C.C. problem!

- Is it a problem?
- Anthropics
- Modifications of gravity and DE. For example is $p = w\rho$ with $w = -1$?

Experimental + Theoretical work

(2) The cosmic microwave background

$T_0 = 2.725$ K

$(\sim 6 \times 10^{-4}$ eV $)$

$\rho_\gamma = \sigma T_0^4 = 0.26$ MeV m^{-3}

$\sim 5 \times 10^{-5}$ critical density

$n_{\gamma,0} = 4.1 \times 10^{8}\ m^{-3}$

$\Omega_{bary} \simeq 0.04$

$\Rightarrow \rho_{bary} \simeq 210$ MeV m^{-3}

But baryons (protons and neutrons mass $\sim$ 939 MeV) are heavy

$$n_{bary,0} = \frac{\rho_{bary}}{m_{bary}} \simeq 0.22\ m^{-3}$$

$$\eta = \frac{n_{bary,0}}{n_{\gamma,0}} \simeq 5 \times 10^{-10}$$

Recombination and decoupling

Recombination: baryonic component from ionized to neutral

Photon decoupling: rate of interactions photons with baryon becomes $\lesssim H^{-1}$. Less than one interaction per Hubble time

Fractional ionization $X \equiv \frac{n_p}{n_p + n_H} = \frac{n_p}{n_{bary}} = \frac{n_e}{n_{bary}}$

We assume only hydrogen exists: nucleosynthesis later

$$H + \gamma \rightleftarrows p\, e^- \qquad Q = 13.6 \text{ eV}$$

- Naively recombination takes place when $T \sim 13.6\,eV \sim 60\,000\,K$

But this is not correct: so many photons around that it is enough the high energy tail is energetic enough to ionize atoms

A proper statistical calculation gives: $T_{rec} \simeq 0.3\,eV = \frac{Q}{42}$
$\simeq 3700\,K$

- The fraction of free electrons drop so rapidly that effectively recombination coincide with decoupling

$$z_{dec} \simeq 1100$$

Q

- Another relevant moment in the evolution of the Universe is matter / radiation equality

$$\begin{aligned} \text{radiation} &\propto a^{-4} \\ \text{matter} &\propto a^{-3} \end{aligned} \qquad z_{equality} \simeq 3570$$

- CMB strong support of hot Big Bang

The peaks

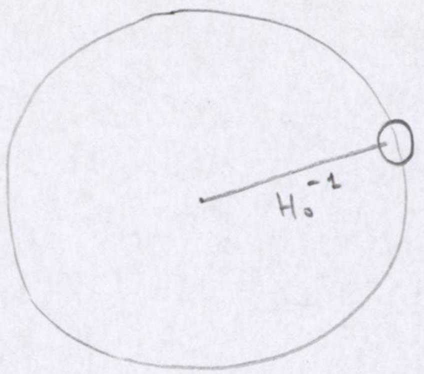
- There is obviously an interesting scale in the problem

At recombination we are in MD. $\rho_{MATTER} \propto a^{-3}$

$H^2 \propto \rho \qquad \frac{H_{rec}^2}{H_0^2} = (1+z)^3 \qquad$ Actually $\frac{H_{rec}^2}{H_0^2} = \Omega_{m;0}(1+z)$

$1+z \propto \frac{1}{a} \qquad H_{rec}^{-1\,now} = \underbrace{(1+z)}_{\text{expanded!}} H_0^{-1} \frac{1}{\sqrt{\Omega_{m;0}^{1/2}(1+z)}}$

$= \frac{1}{\sqrt{\Omega_{m;0}^{1/2}}} H_0^{-1} \frac{1}{\sqrt{1+z}}$



$\theta \sim \frac{1}{\sqrt{\Omega_{m;0}^{1/2}}\sqrt{1+z}} \sim 0.015 \text{ rad} \sim 1^\circ$

- Baryons and photons before decoupling form a fluid with pressure which oscillates

Waves propagate at a speed $< c$ and do not have time to travel to a distance $> H_{rec}^{-1}$.

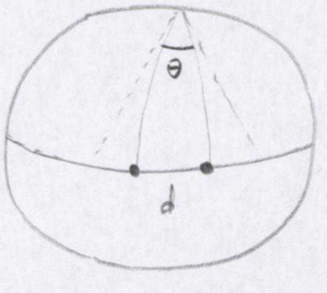
Oscillations in time - oscillations in k

Oscillations only on scales $\lesssim 1^\circ$.

- Why we care so much about CMB?
  - Experimentally clean
  - Simple and computable physics
  - Dependence on cosmological parameters

Examples of parameter dependence:

- Position of the first peak depends on the geometry of the Universe



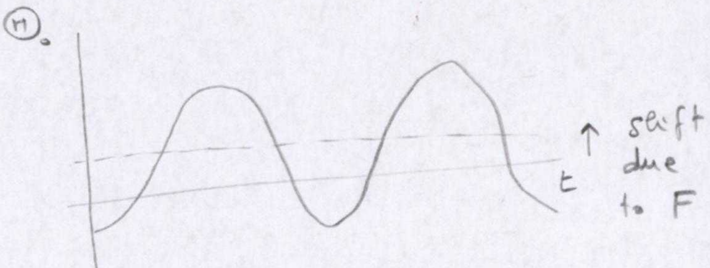
In the presence of curvature the angle corresponding to a given distance changes

For positive curvature, I would interpret as coming from a bigger object

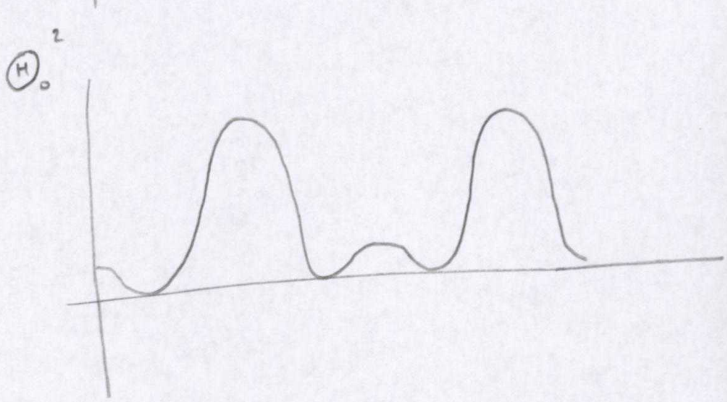
Sub % bound on curvature

- See parameter dependence on webpage

$$\ddot{\Theta}_0 + k^2 c_s^2 \Theta_0 = F$$



More baryons reduce c_s and thus enhance the difference odd-even peaks



Doppler + Damping

(3) Nucleosynthesis

- We cannot see anything beyond LSS : Universe in opaque
- N is the highest energy process we have under full th/exp control

- Nuclear energies ~ MeV   Nucleus: A, Z

(Q)

A: number of nucleons; Z: number of protons

- Binding energy deuterium (^2H)

$$p + n \rightleftarrows D + 2.22\ \text{MeV}$$

Like in recombination we expect nuclei to form when $T \lesssim$ MeV

- $\frac{B_D}{Q} \sim 1.6 \times 10^5$ Since $T_{rec} \sim 3740\ K$

  $T_{nuc} \sim 6\ 10^8\ K$ (~ 300 s)

- Before making nuclei, we must know how many protons and neutrons are around

$T \gg 10$ MeV

$$n\ \nu_e \rightleftarrows p\ e^-$$
$$n\ e^+ \rightleftarrows p\ \bar{\nu}_e$$

Similar to decoupling of neutrinos discussed by Mauro

$$Q_n = m_n c^2 - m_p c^2 = 1.29\ \text{MeV}$$

$$\frac{n_n}{n_p} = e^{-\frac{Q_n}{kT}}$$

Decoupling $T_{freeze} \sim 0.8$ MeV

$$\frac{n_n}{n_p} = e^{-\frac{Q_n}{T_{freeze}}} \simeq 0.2$$

- Nucleosynthesis proceeds step by step. First step is formation of deuterium

$$n\,p \rightleftarrows D\,\gamma$$

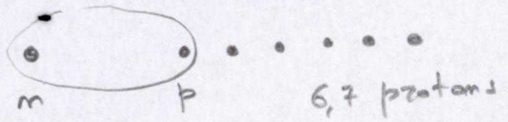
Deuterium bottleneck

- We have to take into account neutron decay! $\tau_n = 890\,s$

$$\frac{n_n}{n_p} \simeq \frac{e^{-200/890}}{5+(1-e^{-200/890})} \simeq \frac{0.8}{5.2} \simeq 0.15$$

- ^4He is very tightly bound. Every neutron will quickly proceed to ^4He

$$Y_p = \frac{\rho(^4He)}{\rho_{bary}} \simeq 0.26$$



$$\frac{2}{7.7} \sim 0.26$$

- Heavier elements are difficult to form:

  T is now low because of D bottleneck

  No stable $A=5$ $A=8$

- η dependence. $\eta\downarrow$ more photons to destroy D and lowering $T_{nuc} \Rightarrow Y_p \downarrow$

  Strong effect on D

- Deuterium observation in low metallicity cloud with Lyα transition ($\neq$ H because of n extra)

- Why baryons and not anti-baryons?

  Baryogenesis   Baryon number violation in BSM?

(4) Inflation

Surprisingly another place that we know very well. A lot we do not know

- Horizon problem: how much can a photon travel?

$$\int_0^{t_0} \frac{dt}{a(t)} = 3t = 2H^{-1}$$

MD: $\frac{da}{dt}\cdot\frac{1}{a} \propto a^{-3/2}$ $\quad a = \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$ $\quad H = \frac{\dot a}{a} = \frac{2}{3}\frac{1}{t}$

Hubble scale is also total space a photon can travel

Why is the CMB temperature isotropic in first approximation?

- Fine tuning problem like hierarchy.

- Curvature problem:

$$H^2 = \frac{8\pi G}{3}\rho + \frac{k}{a^2}$$

curvature is small now, so it was super-small early on: how is it possible?

Solution: period of early acceleration $\ddot a > 0$

- $\int \frac{dt}{a(t)} = \int \frac{da}{\dot a a}$ $\quad \ddot a > 0$ integral diverges in the past!

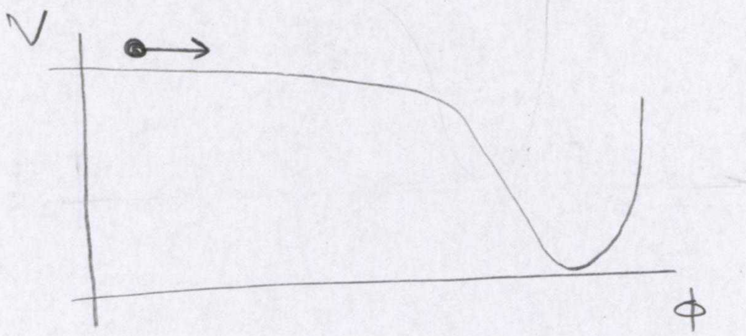
- $\frac{\dot a^2}{a^2} \lesssim \frac{1}{a^2}$ $\quad$ General prediction of flatness

- Another time: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

  But now a Λ is not enough to save the day. Why? Q

- Slow-roll inflation:

  Take a scalar field. What? Higgs Q



$$\ddot{\phi} + \underbrace{3H\dot{\phi}}_{\text{Hubble friction; energy dilutes!}} + V'(\phi) = 0 \quad + \text{ accelerating metric}$$

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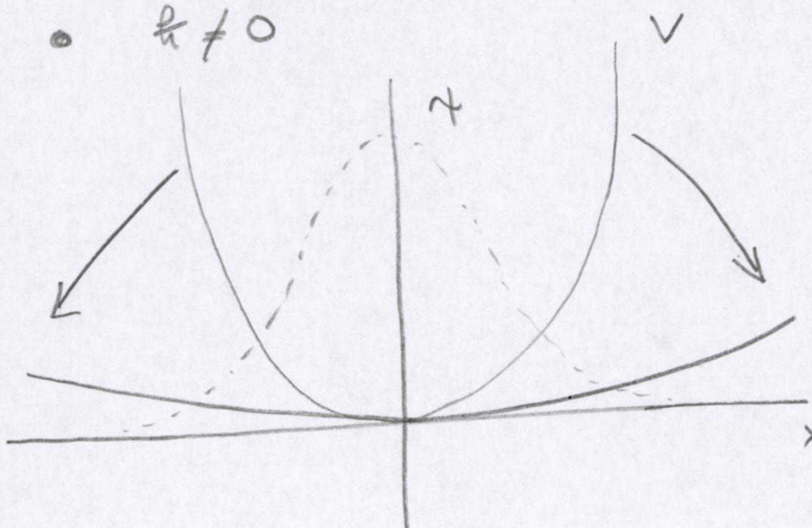
- Reheating

- $\hbar \neq 0$



After the transition
I am not in the vacuum
of QHO anymore

The same happens for each Fourier mode of the inflaton field

- H is $\sim$ constant during inflation

$$a \sim e^{Ht}$$

$\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle \sim$ constant   Each mode has the same variance.

Scale-invariant   Indeed this is what we see in CMB

Not really constant: small variation of the potential energy tilt

- Also the graviton is present and it gets the same kind of perturbations

$\langle \gamma_{\vec{k}} \gamma_{\vec{k}} \rangle \sim$ constant $\longrightarrow$ B-modes

(different normalization, not exactly known)