Quantum Black Holes

*An **IR** Window into **UV** physics*

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Kerr Conference

Albert Einstein Institute

Potsdam, July 2013

- A black hole is at once the ***most simple*** and yet the ***most complex*** object.
- Understanding the ***simplicity*** is in the realm of ***classical gravity*** and understanding the ***complexity*** is in the realm of ***quantum gravity.***

A great deal of **quantitative** information about semiclassical and quantum properties of a black hole has been obtained entirely on the strength of theoretical considerations. Makes for an interesting study in history of science.

Outline

- Classical black holes
- Semi-classical black holes
- Quantum black holes

Quantum black holes provide us with an invaluable tool to learn about the *short distance (UV) structure* of quantum gravity by studying *long distance (IR) properties.*

Classical Black Holes

- ***No hair theorem***:

  A black hole is completely specified by mass, spin, charge much like an elementary particle.

  Kerr-Newman Metric.

  A black hole (unlike a star) is simple!

- ***Event Horizon:***

  A one way surface that ***causally*** separates the outside from the inside of the black hole.

Paradox I

- What happens if you throw a bucket of hot water into a black hole? The entropy of the world outside the black hole would decrease, violating the second law of thermodynamics.

Bekenstein

- ***Resolution**: Second law can be saved if the black hole also has entropy. Then the total entropy of black hole + bucket can increase in accordance with the second law.*

Paradox II

- If a black hole has entropy and mass then by ***first*** law of thermodynamics, it must also have temperature. But then it must radiate which is impossible for a classical black hole because the event horizon is a one way surface.
- **Resolution**: Because of quantum pair creation near the event horizon, a black hole can radiate. Metric is still treated classically.

Hawking

Semi-classical Black Holes

- A black hole has temperature and entropy

$$S = \frac{Ac^3}{4\hbar G} = \frac{A}{4l^2}$$

- This `Bekenstein-Hawking' area formula is remarkably general and involves all three fundamental constants of nature. Enormous entropy signifying a huge complexity.

Paradox III

- If a black hole has entropy then in quantum theory it must be an ensemble of microstates according to Boltzmann. How do we associate so many microstates with a hole in spacetime?
- To resolve this paradox we really need a quantum theory of gravity with a well-defined quantum Hilbert space.

Partially understood in string theory.

Quantum Gravity

- One of the enduring challenges of theoretical physics is to find a consistent framework for Quantum Gravity that unifies General Relativity with Quantum Mechanics.
- String theory offers a promising route towards such a Quantum Theory of Gravity:

  perturbatively UV finite, strong-weak dualities, AdS/CFT holography…

  However…..

Hurdles for String Theory

- We do not have a microscope like a super-LHC to probe the theory directly at Planck scale.
- We do not even know which phase or `compactification' of the theory may correspond to the real world.

How can we be sure that string theory is the right approach to quantum gravity in the absence of direct experiments?

How can we proceed?

- In a such a situation, a good strategy is to focus on **universal** features that must hold in **all phases** of the theory. Analogy with water.
- Entropy of a black hole is one such quantity which gives very precise quantitative thermodynamic information.
- In a quantum gravity, it should be possible to interpret a black hole as an ensemble of states in the Hilbert space of the theory.

Micro from Macro

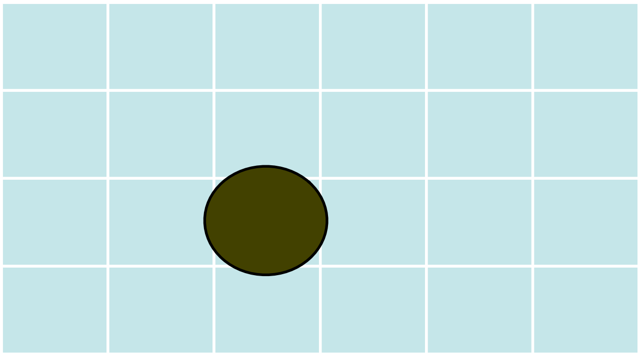
- In the absence of a microscope, one can often learn a lot about the microstructure from thermodynamic properties.
- For example, temperature dependence of specific heat of a metal tells you whether phonons or electrons are the relevant degrees of freedom.

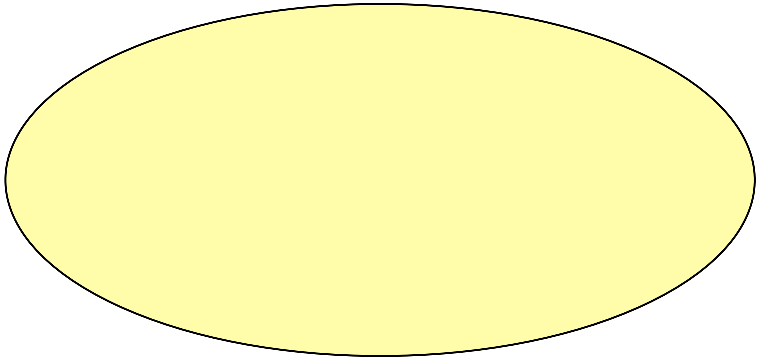
Quantum properties of black holes can be put to good use in an analogous fashion.

Historical Analogy

- Kinetic theory of gases was a triumph of 19th century physics that formed the basis for the atomic hypothesis & later for quantum theory.
- It started with the attempts to explain **macroscopic** properties of ideal gases in terms of **microscopic `atoms'** even though there were no microscopes at the time that could establish the reality of atoms directly.

Entropy of dilute Nitrogen gas





d(N) is the number of ways N molecules of size λ (de Broglie wavelength) can be distributed in volume V. **Microscopic counting** explains **Macroscopic Entropy!**

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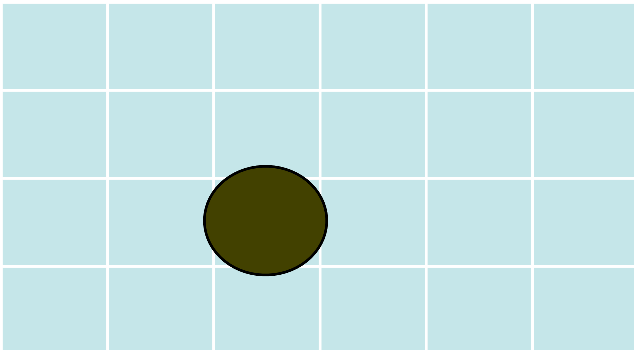

$$\Omega = \left(\frac{V}{\lambda^3}\right)$$

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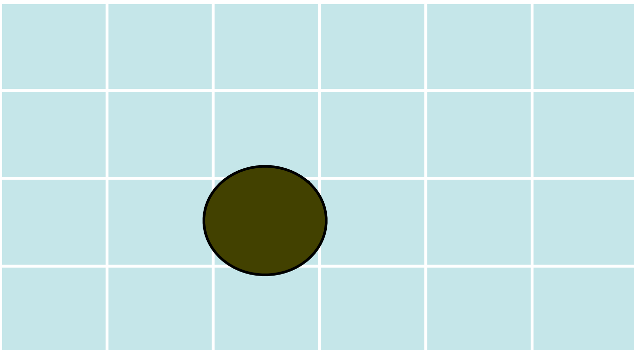

$$\Omega = \left(\frac{V}{\lambda^3}\right)^N$$

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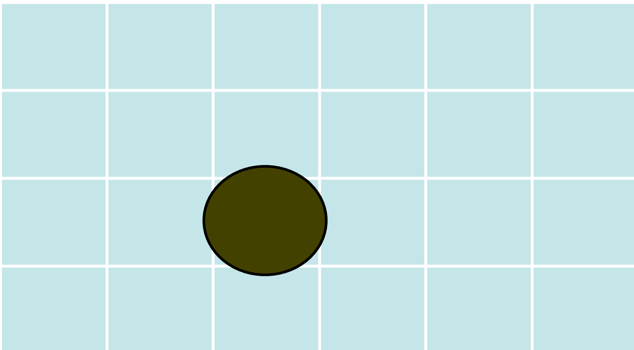
Entropy of dilute Nitrogen gas



$$\Omega = \frac{1}{N!}\left(\frac{V}{\lambda^3}\right)^N$$

d(N) is the number of ways N molecules of size λ (de Broglie wavelength) can be distributed in volume V. **Microscopic counting** explains **Macroscopic Entropy!**

This line of reasoning has already led to some important advances in the 1990s.

- A large class of supersymmetric (charged) black holes can indeed be interpreted as ensembles of microstates as required by Boltzmann.
- Study of black branes (extended versions of black holes) led to holographic equivalence between a theory with gravity (AdS) and a theory without gravity (CFT).

Mostly for large area or large charge.

Black Hole as an Ensemble

- Does this entropy satisfy Boltzmann relation?

$$S = \log(d)$$

- Yes! For example, for a susy black hole with three charges Q_1, Q_2, Q_3 (all large)

$$\frac{A(Q_1, Q_2, Q_3)}{4} = 2\pi\sqrt{Q_1 Q_2 Q_3}$$

Macroscopic *Microscopic*

Bekenstein-Hawking *Strominger-Vafa*

A black hole is simple not because it is like an elementary particle but rather because it is like a thermodynamic ensemble.

This explains why it is both simple & complex!

Can we mine further this very important clue about quantum gravity?

Quantum Black Holes

Given such a beautiful approximate agreement,

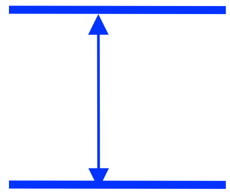
it is natural ask:

- *What **exact** formula does it approximate? What is a **quantum generalization** of the Bekenstein-Hawking area formula for entropy?*
- *How to **systematically compute** the corrections to this quantum entropy and compare with microscopic counting?*

More subtle statistical considerations can yield more interesting information. Let us return to our historical analogy.

For example, we need the N! in our counting because all molecules of nitrogen molecules are identical. *Gibbs* deduced this important fact about the **microstructure** from extensivity of **macroscopic** entropy decades before the spin-statistics theorem in QFT.

- Classical equipartition theorem gives wrong specific heat for nitrogen. Because vibrational degrees are frozen at low temperature



- **Maxwell** regarded this as the *`greatest difficulty of classical molecular theory'* as early as **1859**. **Jeans** made a prescient remark in **1890** that somehow *`the degrees of freedom seem to be frozen.'*
- Serious crisis of classical physics.

What is new?

- Much of earlier work on black holes and holography is for black holes with large area.
- Our focus here will be on **finite size (finite charge) corrections to the black hole entropy**.
- Unlike the leading Bekenstein-Hawking formula, these **depend** sensitively on the phase under consideration & provide a useful window into the **UV structure** of the theory.

Why obsess with black holes?

- *Universal and extremely stringent constraint*

  In **any** phase of the theory that admits **any** black hole as a solution, it should be possible to view it as an ensemble of quantum states.

- *Quantum Entropy as **IR** window into the **UV***

  Finite size corrections to quantum entropy of a black hole give very precise quantitative information about the UV of the theory.

Quantum Entropy

- We are looking for a generalization of sort

$$S = a_0 A + a_1 \log(A) + a_2 \frac{1}{A^2} + \ldots + c_1(A) e^{-A} \ldots$$

- There is a generalization due to Wald but it is applicable only for local actions. Massless fields in loops will contribute nonlocal terms.
- We need a notion of quantum entropy that includes nonlocal quantum effects as well.
- *A proper definition became available only in 2008 for susy black holes using AdS/CFT holography.*

Holography

- **Paradox IV**: If the entropy in a volume of space scales with volume then the spontaneous process of collapse into a black hole would decrease entropy violating second law. Unless the initial entropy scales with area and not volume.
- A (d+1)-dimensional theory must have the degrees of freedom of a d-dimensional theory like the holographic imprint of a 3d object onto a 2d hologram.

Holography in String Theory

- This heuristic idea is realized concretely in string theory in the AdS/CFT correspondence. Maldacena
- A remarkable quantum equivalence between

a theory ***with*** gravity **AdS**

(strings moving in Anti de Sitter spacetime)

&

and a theory ***without*** gravity **CFT**

(Conformal Field Theory in one less dimension).

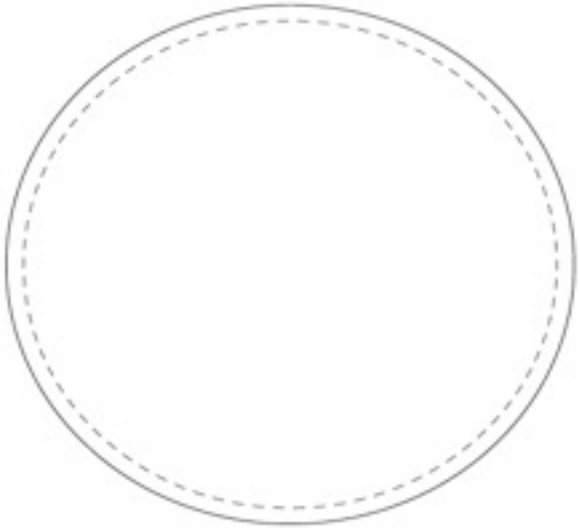
AdS_{p+2}/CFT_{p+1}

- Near horizon limit of a ***p-brane*** geometry is ***Anti de Sitter spacetime AdS$_{p+2}$*** .
- Low energy modes of the p-brane are desribed by ***Conformal Field Theory CFT$_{p+1}$***

  Renormalization scale of the CFT becomes the additional dimension. Even though there is much evidence for this equivalence, its full implications for quantum gravity are far from being understood.

AdS_2/CFT_1 and Quantum Entropy

- The near horizon geometry of a extremal charged (non-spinning) black hole is AdS_2 .
- Quantum entropy can then be defined as a functional integral *W(Q)* in AdS_2 *over all string fields with appropriate boundary conditions, operator insertion, and a renormalization procedure*. ***Sen***
- For large charges, logarithm of *W(Q)* reduces to Bekenstein-Hawking-Wald entropy.



- Euclidean *AdS_2* space is a disk with a metric

$$ds^2 = (r^2 - 1)d\theta^2 + \frac{dr^2}{r^2 - 1}$$

- Put a cutoff at $r = r_0$

W(Q)	d(Q)
Black Hole (charge Q)	Brane (charge Q)
Quantum Entropy	Counting of States
AdS_2	CFT_1
Spacetime Geometry	Hilbert Space

This gives the proper quantum generalization of

Bekenstein-Hawking ---- *Boltzmann*

Can we compute both sides?

How to evaluate it?

- *d(Q)* is the number of bound states of a brane system which is an extremely difficult dynamical problem in general.
- *W(Q)* is given as a formal functional integral and it is far from clear what to do with it.

A lot of progress has been made in several models in these computations. I will describe a couple of simpler examples for illustration.

Computing d(Q)

- For a large class of models this problem has been solved through the work of many people over several years.
- For a black hole with electric charge vector *q* and magnetic charge vector *p*, the degeneracy often depends only on a few duality invariants.
- Degeneracy given in terms of Fourier coefficients of ***modular forms***.

Modular forms

- A holomorphic function $F(\tau)$ on the upper half complex plane is a modular form of weight k, if it transforms as

$$F\left(\frac{a\tau + b}{c\tau + d}\right) = (c\tau + d)^k F(\tau)$$

for *a, b, c, d, k* integers and *ad-bc =1*

The matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ *form the group SL(2, Z) under matrix multiplication.*

Electric States in Heterotic String

- Degeneracy *d(q)* depends not only on the duality invariant $q^2/2 = N$. It is given by

$$F(\tau) = \frac{1}{q \prod_n (1 - q^n)^{24}}, \qquad q = e^{2\pi i \tau}$$

$$= \sum_n c(n) q^n$$

$$d(N) = c(N).$$

Here $F(\tau)$ is a modular form of weight -12.

Computing W(Q)

This is even harder so you need clever ideas.

- **First pass**: One charge large, all other finite.

  Use scaling, anomaly in-flow, Chern-Simons terms in the effective action.

- **Second pass**: All charges finite

  Use 'localization' to reduce a supergravity path-integral onto finite-dimensional manifold in field space of instanton solutions.

Exact results with one charge large

- A 5d-black hole with spin and three charges.

log (W) matches beautifully with *log (d)*.

$$\pi\sqrt{4Q_1(Q_2+3)\left(Q_3-\frac{J^2}{Q_2(Q_1-1)}\right)}$$

- *Exact result* in the limit of Q_3 large.
- Depends on specific higher derivative corrections in the effective action of string theory with *precise* coefficients from ***UV***.

All charges finite

- When all charges are finite, we have to evaluate the path integral. Looks hopeless, even foolish. But one can go quite far using the localization technique for supersymmetric path integrals.
- Localization is a tool that has been used extensively in quantum field theories to make many nontrivial computations (indices, Wilson loops…). This is a first application in a gravitational context

Localizing Instantons

- The functional integral localizes onto a submanifold in field space onto which Q vanishes = manifold of localizing instantons.
- We found new localizing instantons in AdS_2 for supergravity coupled to ***r*** vector multiplets
- It is possible to evaluate also the induced measure and renormalized action.
- Long and difficult computations but we have something to compare with, which helps.

Final integral

Using an N=2 truncation and keeping only vectors and F-terms the *functional* integral reduces to a single *ordinary* integral

$$W(\Delta) = 2\pi(\frac{\pi}{\Delta})^{7/2}\frac{1}{2\pi i}\int \frac{d\sigma}{\sigma^{9/2}}\exp[\sigma + \frac{z^2}{4\sigma}]$$

The contour is parallel to imaginary axis.This gives a modified Bessel function

$$W(\Delta) = \tilde{I}_{7/2}(\Delta)$$

Degeneracy, Quantum Entropy, Wald Entropy

Δ	$C(\Delta)$	$W_1(\Delta)$	$\exp(\pi\sqrt{\Delta})$
3	8	7.972	230.765
4	-12	12.201	535.492
7	39	38.986	4071.93
8	-56	55.721	7228.35
11	152	152.041	22506.
12	-208	208.455	53252.
15	513	512.958	192401

Positivity

- Note that $C(\Delta)$ are alternating in sign so that $d(\Delta) = (-1)^{\Delta+1} C(\Delta)$ is strictly positive.
- Surprising for a quantum field theorist or for a number theorist, because Fourier coefficients of a modular form do not *a priori* have any positivity property.
- *This is a **prediction from IR** quantum gravity for black holes which is **borne out by the UV**.*

Integrality

- The area of the black hole horizon is $4\pi\sqrt{\Delta}$. *The Bessel function sums up an infinite series of perturbative corrections in inverse powers of area.* Remarkably, the functional integral quantum entropy gives an answer that is very close to the integral degeneracy.
- *By contrast, the exponential of Wald entropy is very far from the integer.* Even for $\Delta = 15$

when area is large, the nonlocal quantum corrections make a substantial contribution.

Hardy- Ramanujan-Rademacher

- There are subleading nonperturbative corrections. Because of modular symmetry the degeneracy admits a remarkable expansion

$$d_c(\Delta) = \sum_{c=1}^{\infty} \tilde{I}_{\frac{7}{2}}\left(\frac{\pi\sqrt{\Delta}}{c}\right) \frac{1}{c^{9/2}} K_c(\Delta)\,.$$

- Here K_c are some nontrivial number theoretic phases. Can we get this whole structure?

Nonperturbative contributions

- Euclidean AdS_2 is a disk and admits Z_c orbifolds by a rotation through $2\pi/c$. The metric is such that this does not change the boundary conditions.
- The orbifolds admit localizing instantons whose action is down by a factor of c, so that explains the whole series. The phases arise from Wilson line insertions. So we are very close to the anser but have not yet been able to obtain precisely K_c

Summary

- Black holes continue to be an important source of new theoretical ideas. Because their thermodynamic properties are deduced from robust and well-tested physical principles, it remains our most reliable guide in the search for a coherent framework for quantum gravity.
- String theory is an extremely rigid theoretical structure and seems capable of explaining a black hole as an ensemble in a quantum Hilbert space.
- It is remarkable that gravity can 'see' the integrality of a non-perturbative count of quantum states.

- ***Dabholkar, Gomes, Murthy, Sen***

  1009.0256
- ***Dabholkar, Gomes, Murthy***

  1012.0265, 1111.1161,
- ***Dabholkar, Murthy, Zagier***

  1208.4074

Earlier work with

Denef, Gaiotto, Guica, Iizuka, Iqubal, Kallosh, Maloney, Moore, Nampuri, Narayan, Pioline, Shigemori, Trivedi