PROBLEMS of the BB MODEL - Lecture 1

Introduction: BB model has some problems of initial conditions that must be fine tuned to recover the Universe we observe today. We would like to have a theory were such a problem does not exists.
Inflation solves elegantly ~~of~~ all the fine tuning problems of the BB model. Moreover gives a mechanism that explain the origin of the structures in the Universe.
During inflation indeed quantum fluctuations are generated, seeding the Universe of small inhomogeneities which then originated the LSS.

Goal: How inflation solves the major problems of the BB model while quantum mechanically providing a mechanism to generate the primordial seeds for the LSS of the Universe.

FRW COSMOLOGY

Modern cosmology is based on 2 observational facts:

i) the Universe is expanding

ii) on very large scales ($\gtrsim 100$ Mpc) the matter distribution is homogeneous and isotropic.

Then the average space time (i.e. at those scales) is well described by the FRW metric (hom and isotropic solution of E eqs.):

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right) \qquad \overset{c=1}{d\Omega^2}$$

$\hookrightarrow$ K: curvature: $\pm 1, 0$ curvature of the 3-dim spatial hypersurfaces.

iii) Another experimental fact: $K \to 0$ $\Rightarrow$ Restrict to $K=0$ case!

r: dimension less
K: real parameter, can be always chosen to be $\{-1,0,1\}$ redefining $a(t)$

$\hookrightarrow$

In this case E.E. gives the so called Friedmann equations, that give the evolution of the scale factor:

$$\begin{cases} H^2 = \dfrac{\rho}{3 M_{pe}^2} \\ \dot{H} + H^2 = -\dfrac{1}{6 M_{pe}^2}(\rho + 3p) \end{cases} \qquad M_{pe} = \frac{1}{\sqrt{8\pi G}}, \quad c = \hbar = 1$$

ρ, p : energy density and pressure of the background stress-energy tensor
We assumed a perfect fluid!

~~baro~~ barothropic fluid: $p = w\rho$
ordinary matter : $w = 0$
radiation : $w = 1/3$
$\Big\}$ obey SEC

Causal structure of FRW spacetime

Better to define the conformal time τ: $d\tau = dt/a$.
Then the metric reduces:

$$ds^2 = a^2(\tau)\,\eta_{\mu\nu}\,dx^\mu dx^\nu$$

$\Rightarrow$ The causal structure is the same as Minkosky:
radial prop. of photons: $0 = ds^2 = a^2(\tau)\left(-d\tau^2 + dr^2\right)$

$r(\tau) = \pm\tau + \text{const}$
light rays propagates on 45° lines in the $\tau - r$ plane.

Given t_1 and t_2 the maximum distance travelled by a photon is

$$\Delta r = \Delta\tau = \int_{t_1}^{t_2} \frac{dt'}{a(t')}$$

Now solving F. eqs. for a fluid with $w > -1/3$ we find:

$$a(t) = a_0\, t^{\frac{2}{3(1+w)}}$$

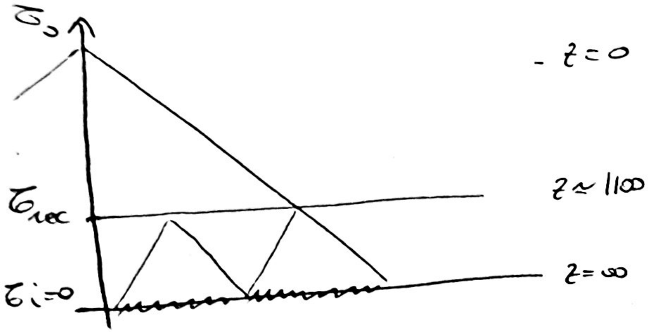
Notice that there is a time in which $a(t_i) = 0$!

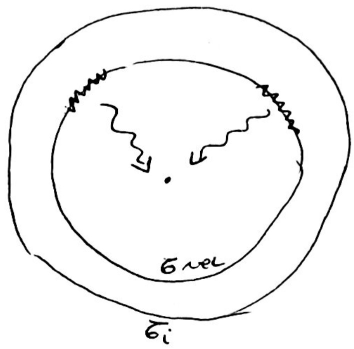
→ <u>Comoving particle horizon</u>

Maximal distance travelled by a photon since the begin of times:

$$d_{Hor} = \Delta r \equiv \tau - \tau_i = \int_{t_i}^{t} \frac{dt'}{a(t')} \qquad \dot{a} = \frac{da}{dt} \Rightarrow da = \dot{a}\,dt$$

Photons travelled a finite distance from the beginning of the Universe!



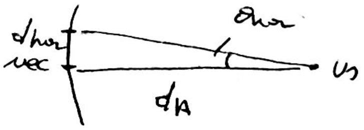


<u>Horizon problem</u>

Exercise: how many disconnected patches in the sky?

- $d_{Hor} = \tau_{rec} - \tau_i$
- $d_A = \tau_0 - \tau_{rec}$: comoving angular distance from us

$$\theta_{hor} \equiv \frac{d_{hor}}{d_A}$$: Comoving angular diameter distance from us to recombination



Now $$\Delta\tau = \int_{t_1}^{t_2} \frac{dt'}{a(t')} = \int_{a_1}^{a_2} \frac{da}{H a^2} = -\int_{z_1}^{z_2} \frac{dz}{H(z)}$$

$dt = \frac{da}{\dot{a}}$ $\qquad z = \frac{1}{a} - 1$

For a Universe filled with matter, radiation and Λ:

$$H(z) = H_0 \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{\Lambda,0}}$$

~~[illegible]~~

: Find θ_{hor} doing numerically the integral ; using $\Omega_{m,0} = 0.3$, ~~[illegible]~~

$\Omega_{r,0} = \frac{\Omega_m}{(1+z_{eq})}$, $z_{eq} = 3400$

$\rightarrow$ $\underline{\theta_{hor} = 1.16°}$ $(2 \cdot 10^{-2}\ \text{rad})$: patches that subtend an angular diameter of $\sqrt{2}\,\theta_{hor}$ are causally disconnected among themselves

Solid angle subtended by the patch : $\Omega \propto \frac{\text{Area}}{\text{distance}^2} = \frac{\pi d_{hor}^2}{d_A^2} = \pi \theta_{hor}^2$

$$\# = \frac{4\pi}{\pi \theta_{hor}^2} \simeq \underline{\underline{10^4}}$$

<u>But the CMB has no patches!</u>

<u>Flatness</u> problem

Why the universe is so flat? $\rightarrow$ Requires fine tuning!

<u>Fine tuning</u> : initial values must be set with very high precision in order to reproduce the observed quantities

In this case what we need very small is Ω_K in the early universe.

Fried eqs with curvature : $\underline{H^2 = \frac{\rho}{3 M_{pl}^2} - \frac{K}{a^2}}$

If $K = 0 \Rightarrow \rho = \rho_{crit} \equiv 3 M_{pl}^2 H^2$

Dividing by H^2

$$1 = \Omega_{tot} - \frac{K}{a^2 H^2} \longrightarrow \boxed{\frac{K}{a^2 H^2} = \Omega_{tot} - 1}$$

Current measurement (PLANCK 2015) $\Omega_K \equiv 1 - \Omega = 0.005 \pm 0.016$

Why Ω_K is so small? Was it bigger/smaller in the past? Show that:

$$\frac{d\Omega}{d\ln a} = (1+3w)\,\Omega\,(\Omega-1)$$

Proof: $\Omega - 1 = \frac{K}{a^2H^2} \rightarrow \frac{d\Omega}{da} = -\frac{2K}{a^2H^2}\underbrace{\left(\frac{1}{a}+\frac{1}{H}\frac{dH}{da}\right)}_{(*)}$

Now: $\frac{dH}{da} = \frac{dH}{dt}\frac{dt}{da} = \frac{\dot H}{Ha}$

$(*) = \frac{1}{a}\left(1+\frac{\dot H}{H^2}\right) = \frac{1}{aH^2}(H^2+\dot H) = \frac{-1}{aH^2}\frac{\rho(1+3w)}{6M_{pe}^2} = -\frac{1}{2a}\Omega(1+3w)$

then: $\frac{d\Omega}{d\ln a} = -8\underbrace{\frac{K}{a^2H^2}}_{\Omega-1}\cdot\left(-\frac{1}{8}\right)\Omega(1-3w)$

The evolution equation for Ω shows that $\Omega=1$ is unstable for ordinary matter!

For $(1+3w)>0$: $\Omega>1 \Rightarrow \frac{d\ln\Omega}{d\ln a}>1 \rightarrow \Omega$ increases in time

$\Omega<1 \Rightarrow \frac{d\Omega}{d\ln a}<1 \rightarrow \Omega$ decreases in time

To reach the current bound on Ω_K the universe in the past had to be extremely flat!

<u>Inflationary solution to the horizon and flatness problems</u>

- <u>Horizon problem</u>

go back to $\tau = \int\frac{dt'}{a(t')} = \int\frac{dt'}{da}\frac{da}{a} = \int d\ln a\left(\frac{1}{aH}\right)$ → comoving H radius

So the conformal time elapsed between a_1 and a_2 depends on the comoving Hubble radius $1/aH$. For a barotropic fluid we have: $\frac{1}{aH} = a^{\frac{1}{2}(1+3w)}$

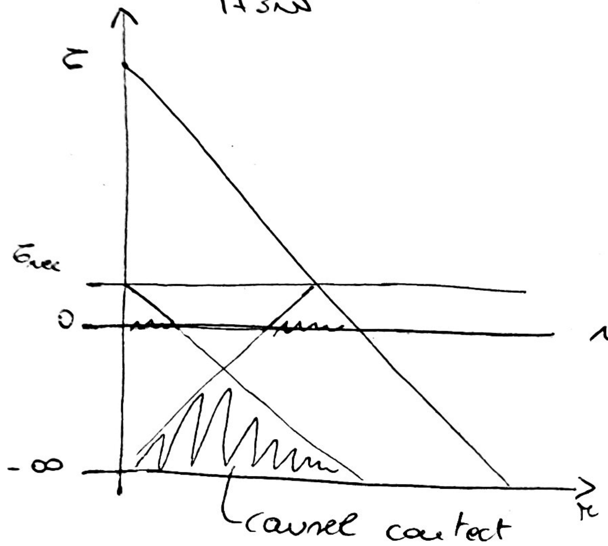
For ordinary matter (obeys SEC) $1+3w>0 \Rightarrow \frac{1}{aH}$ is always ~~decreasing~~ increasing

(SEC: $(T_{ab} - \frac{1}{2} T \delta_{ab}) x^a x^b \geq 0$, x^a: timelike)

With this $\tau = \int d\log a \, \frac{1}{Ha} = \frac{2}{1+3w} a^{\frac{1}{2}(1+3w)}$

So for conventional matter $a=0 \Rightarrow \tau_i = 0$. If instead we consider a fluid such that $w < -1/3$ we see that:

$\tau_i \simeq \frac{2}{1+3w} a^{1/2(1+3w)} \longrightarrow -\infty$!



Conformal diagram of inflationary cosmology

If inflation happened $\tau = 0$ is not the initial time. There is now much more time to make the Universe homogeneous.

Horizon problem

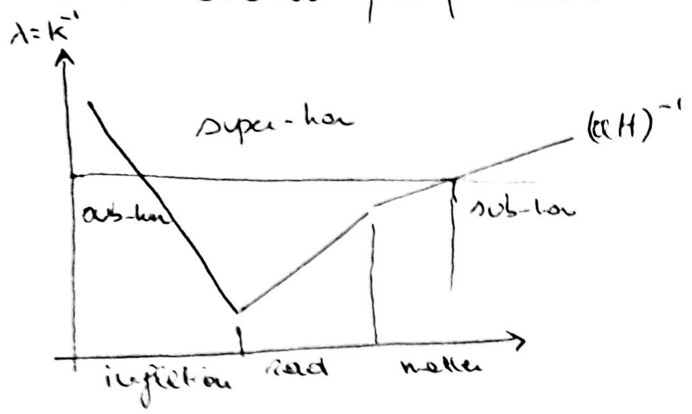
Consider again the evolution equation for Ω.

$$\frac{d\Omega}{d\log a} = (1+3w)\,\Omega\,(\Omega-1)$$

during inflation $(1+3w)$ is negative and therefore $\Omega = 1$ becomes an attractor! The system evolves towards $\Omega = 1$

$\Rightarrow \Omega_K \to 0$ no matter how large it is.

Also another perspective:



Definitions of inflation

Period of shrinking of the Hubble radius

$$\frac{d}{dt}(aH)^{-1} < 0 \implies [(aH)^{-1}]^{\cdot} = (\dot{a}^{-1})^{\cdot} = -\dot{a}^{-2}\ddot{a} = -\frac{\ddot{a}}{\dot{a}^2} < 0 \iff \underline{\ddot{a} > 0}$$

• During inflation the universe is expanding in an accelerated way

↳ other possible def of inflation.

Alternatively:

$$\frac{d}{dt}(aH)^{-1} = -\frac{\dot{a}}{a^2 H} - \frac{\dot{H}}{aH^2} = -\frac{1}{a^2H^2}(\dot{a}H + a\dot{H}) = -\frac{1}{a}\left(1 + \frac{\dot{H}}{H^2}\right) < 0$$

$$\implies \underline{\epsilon \equiv -\frac{\dot{H}}{H^2} < 1} \qquad \text{where } \epsilon > 0$$

Conditions for inflation

$\epsilon < 1 \implies$ During inflation H does not change much in an Hubble time (characteristic time scale of inflation)

$\sim$ almost exponential expansion!

Moreover we need inflation to last long enough to make in c.e. el contact the largest scales we observe today. This means that ϵ cannot vary to much in a Hubble time

$\Rightarrow \eta \equiv \frac{\dot{\epsilon}}{\epsilon} \cdot \frac{1}{H} < 1$ $\qquad\qquad t_H = 1/H$

How can we quantify how the "duration" of inflation?

Define: e-folds # : $dN = d\log a = H dt$ $\qquad \frac{da}{dt} = \dot{a}$

~~Suppose H = const => a = ...~~

~~$N_{tot} = \log\left(\frac{a_f}{a_i}\right)$~~

$$N = \int_{a_i}^{a} d\log a = \log\left(\frac{a}{a_i}\right)$$ $\to$ Measure of how the scale factor have increased

To solve the horizon problem on the CMB scales we need $N_{tot} \sim 40 \div 60$

Suppose $H = \text{const} \Rightarrow a = a_I e^{Ht} = a_I e^N$ then $a_F = a_I e^{40} \sim 10^{17} a_I$!

Huge expansion!

$\hookrightarrow$ That's we need $|\eta| < 1$!

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Why N_{tot} is not fixed?

It comes from demanding $\left.\frac{1}{(aH)}\right|_{infl} \geq \left.\frac{1}{(aH)}\right|_{now}$

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