Cosmic Microwave Background: anisotropies and fundamentals

Giulio Fabbian



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Petnica Cosmology Summer school 2017

Outline

- Few words on some basic concepts:
 - What is CMB
 - What are CMB anisotropies
 - CMB polarization
- Few very recent research topics/findings:
 - CMB polarization experiments
 - CMB lensing
 - CMB-LSS correlation
- Data analysis in cosmology: CMB examples and beyond



The Universe and the CMB



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The CMB radiation: a bit of history

Penzias & Wilson, 1964



The CMB radiation: a bit of history

Gamow & Alpher



Penzias & Wilson, 1964



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Penzias & Wilson, 1964





Nuove misure, nuovi problemi

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Nuove misure, nuovi problemi





6



6



Spherical harmonics (caveat: n=l)







CMB fundamentals - PSI Cosmology 2017

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Our goals: understand this figure





The second peak



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The other peaks



The other peaks



Radiation driving



Radiation driving



Radiation driving

- Gravitational potentials in radiation dominated era decay with time
- When fluids inverts oscillations there is no potential well: larger rarefaction amplitudes, information on matter-radiation equality



Total contributions



Baryon effects

- Enhance compression and shift 0-point of oscillations
- Slow down sound speed and change sound horizon



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Dark matter effects

• No baryons = no acoustic oscillations


Dark matter effects

• No baryons = no acoustic oscillations



Dark Energy effects



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The power spectrum of temperature fluctuations in the Cosmic Microwave Background



planck



The power spectrum of temperature fluctuations in the Cosmic Microwave Background



planck





The cosmic pie in 2017



- Detection of scalar spectral: confirm one prediction of inflation
- Reconcile astrophysical data of galaxy + star formation

Polarization of the CMB



Polarization of the CMB



Polarization of the CMB



Why it is useful?

History of galaxy formation through gravitational lensing

First formation of stars



Inflation physics



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Stokes parameters and polarization

• A polarized field is defined by its Stokes parameters:

 $E_{x} = a_{x} \cos \left[\omega_{0} t - \theta_{x}(t)\right] \qquad E_{y} = a_{y} \cos \left[\omega_{0} t - \theta_{y}(t)\right]$

- $I \equiv \left\langle a_{x}^{2} \right\rangle + \left\langle a_{y}^{2} \right\rangle$ $Q \equiv \left\langle a_{x}^{2} \right\rangle \left\langle a_{y}^{2} \right\rangle$
- $\mathbf{U} \equiv \langle 2\mathbf{a}_{\mathbf{x}} \mathbf{a}_{\mathbf{y}} \cos(\mathbf{\theta}_{\mathbf{x}} \mathbf{\theta}_{\mathbf{y}}) \rangle$
- $V \equiv \langle 2a_{x}a_{y}\sin(\theta_{x}-\theta_{y}) \rangle$



 Polarization is a headless vector, identical to itself for right-handed rotation of 180deg: it's a spin-2 complex field

 $Q \pm iU \rightarrow (Q \pm iU)e^{\mp 2i\psi} \Rightarrow Q + iU$ is spin -2

 Circular polarization not produced by Thomson scattering, let's neglect it for the time being...

Spin-weighted spherical harmonics

• Generalization of scalar spherical harmonics for spin fields:

$${}_{s}Y_{l,m}(\theta,\phi) = e^{im\phi} \left[\frac{(l+m)!(l-m)!}{(l+s)!(l-s)!} \frac{2l+1}{4\pi} \right]^{\frac{1}{2}} \sin^{2l}(\theta/2)$$
$$\times \sum_{r} \binom{l-s}{r} \binom{l+s}{r+s-m} (-1)^{l-r-s+m} \cot^{2r+s-m}(\theta/2).$$

 Can be related to scalar harmonics through spin raising/lowering operators (essential for calculations!)

$$\overline{\partial}_{s} f(\theta, \phi) = -\sin^{s} \theta \left[\frac{\partial}{\partial \theta} + i \csc \theta \frac{\partial}{\partial \phi} \right] \sin^{-s} \theta_{s} f(\theta, \phi)$$

$$\overline{\partial}_{s} f(\theta, \phi) = -\sin^{-s} \theta \left[\frac{\partial}{\partial \theta} - i \csc \theta \frac{\partial}{\partial \phi} \right] \sin^{s} \theta_{s} f(\theta, \phi)$$

$$\begin{aligned} \eth_{s} Y_{l,m}(\theta, \phi) &= \sqrt{(l-s)(l+s+1)_{s+1}} Y_{l,m}(\theta, \phi) \\ \bar{\eth}_{s} Y_{l,m}(\theta, \phi) &= -\sqrt{(l+s)(l-s+1)}_{s-1} Y_{l,m}(\theta, \phi) \end{aligned}$$

E/B decomposition

• Coordinate independent decomposition of spin-fields in the harmonic domain

$$(Q + iU)(\theta, \phi) = \sum_{2} a_{\ell m 2} Y_{l,m}(\theta, \phi) \longrightarrow sE_{\ell m} = sa_{\ell m}^{E} = \frac{sa_{\ell m} + -sa_{\ell m}}{2}$$
$$(Q - iU)(\theta, \phi) = \sum_{-2} a_{\ell m - 2} Y_{l,m}(\theta, \phi) \longrightarrow sB_{\ell m} = sa_{\ell m}^{B} = i\frac{sa_{\ell m} - -sa_{\ell m}}{2}$$

• Non-local quantity, analogy with decomposition of electromagnetic field



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Non-local quantity, analogy with decomposition of electromagnetic field



- Maxwell's equation: waves propagate in the orthogonal w.r.t. their electric field direction
- When a incoming wave scatters, only a given component of the incoming wave is diffuse
- What happens when you have more than a single waves interacting....
- Let's assume we have an isotropic distribution (i.e only monopole)



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• What if we have a dipole anisotropy?



• What if we have a dipole anisotropy?

• What if we have a quadrupolar anisotropy?



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How to generate a quadrupole: scalar perturbations

• Single plane wave of scalar perturbation propagating along z



Polarization modulated by quadrupole orientation and projection effects



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Polarization from scalar perturbation



- Polarization probes mainly baryon velocity at last scattering
- E-modes peak at minima of T (pressure and velocity are out of phase)
- T and E-modes are correlated
- Probe nature of pertubations
- B-modes are generated at second order in perturbation theory or through gravitational lensing

Tensor perturbations

• Transverse-traceless perturbation behave like gravity waves: distortion of space in the plane of the perturbation



- Generation of E and B at the same time and with the same amplitude
- Gravity waves decay once entered in the horizon: contribution confined to large scales

$$\ddot{H}_T^{(\pm 2)} + 2\mathcal{H}\dot{H}_T^{(\pm 2)} + (k^2 + 2K)H_T^{(\pm 2)} = 8\pi G a^2 p \pi_T^{(\pm 2)}$$

Tensor power spectrum r=0.2



Large scale T-E correlation



E (anti) correlation

- Photon fluid flows from hot regions to cold initially
- Around a crest: intensity peaks in the directions along the crest and falls off to the neighboring troughs

B no correlation

- Final polarization is orthogonal to the crest or parallel to the trough for points on the through
- Pattern is tangential around hot spots and radial around cold spots

Real space T-E correlation (scalar)



Credits Anthony Challinor

Real space T-E correlation (tensors)

Tensor E-modes polarization

Tensor B-modes polarization



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Current maps from Planck (satellite)



Current maps from Planck (satellite)



Current maps from Planck (satellite)



Cosmological constraints of CMB polarization

- E-modes, TE correlation sensible to physics of acoustic oscillations
- No Sachs-Wolf, ISW, nor Doppler terms for polarization: signal is cleaner (if noise is sufficiently low)



T/S constraints

• B-modes : smoking-gun evidence for gravitational waves in the early universe

$$\Delta_{s}^{2}(k) \approx A_{s}k^{n_{s}-1} \quad n_{s} = 1 - 6\varepsilon + 2\eta \quad \longrightarrow \quad r \equiv \frac{\Delta_{t}^{2}(k)}{\Delta_{s}^{2}(k)} = 16\varepsilon$$
$$\Delta_{t}^{2}(k) \equiv A_{T}k^{n_{t}} \quad n_{t} \approx -2\varepsilon$$

• Can constraints model and energy scale of inflation $V^{1/4} \approx \left(\frac{r}{0.01}\right)^{1/4} 10^{16} \text{GeV}$



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Reionization



- New stars re-ionize the plasma in the universe
- Rescattering erases fluctuations below the horizon scale and regenerates them on the horizon scale at that time

$$\ell_{re} \approx 2 \frac{(\eta_0 - \eta_{re})}{\eta_{re} - \eta_*}$$

- Change in the overall normalization for T and E at small scales
- Large scale E & B proportional to optical depth
- Limitation due to cosmic variance

Inflationary GW and ground-based detectors



Parity violating physics

- Non-zero TB, EB power spectra
- E.g. chiral gravity different amount of I.h and r.h GWs

$$C_{\ell}^{BB} = \int dk \left(\Delta_{\ell,,\mathrm{T}}^{B}(k,\eta_{0}) \right)^{2} \mathcal{P}_{\mathrm{T}}^{(+)}(k),$$

$$C_{\ell}^{TB} = \int dk \Delta_{\ell,\mathrm{T}}^{T}(k,\eta_{0}) \Delta_{\ell,\mathrm{T}}^{B}(k,\eta_{0}) \mathcal{P}_{\mathrm{T}}^{(-)}(k),$$

$$C_{\ell}^{EB} = \int dk \Delta_{\ell,\mathrm{T}}^{E}(k,\eta_{0}) \Delta_{\ell,\mathrm{T}}^{B}(k,\eta_{0}) \mathcal{P}_{\mathrm{T}}^{(-)}(k),$$

$$\mathcal{P}_{\mathrm{T}}^{(\pm)}(k) = r_{(\pm)} \times \mathcal{A}_{\mathrm{S}} \times \left(\frac{k}{k_{0}}\right)^{n_{\mathrm{T}}}$$

- Primordial magnetic fields
- Another alternative: axion interactions

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \nabla_{\mu} \phi \nabla^{\mu} \phi - V(\phi) \\ &- \frac{g_{\phi}}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}. \end{aligned}$$



The forgotten V

- Thomson scattering does not produce V but does not erase it either
- Extension of standard model of EM interactions or primordial magnetic fields...

of accelenated expansion of accelenated expansion of the physical Hi

$$f(\nu) = 2.79 \times 10^{-12} \left(\frac{B}{\text{green}} \right) \left(\frac{GHz}{\text{tes on the physics of}} \right) (1+z_*)$$

• or by astrophysical magnetic fields

the techniques of [13] (as well as the earlier 1 the light of the considerations developed here $\delta \phi_{FC} \approx 3 \times 10^{-1} (1+z)$ (1+z) is developed here for a comoving magnetic field from 5 to 10 1 for $\ell_{\times} < \int 20^{d\ell}$ ($\frac{1}{1 \text{ kpc}} (200 \text{ km}^{-3}) = 0$) is developed here $\int 10^{2} \text{ km}^{-2}$ (1 + z) is developed here for a comoving magnetic field from 5 to 10 1 for $\ell_{\times} < \int 20^{d\ell}$ ($\frac{1}{1 \text{ kpc}} (200 \text{ km}^{-3}) = 0$) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-2}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-3}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-3}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-3}$ (10 km^{-3}) is developed here $\int 10^{2} \text{ km}^{-3}$ (10 km^{-3}) (



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Is CMB "the" CMB?

- CMB is a snapshot of the universe at z 1100.... plus something else
- CMB gets contaminated by evolving astrophysical objects and foregrounds
- Imprint of ongoing large scale structures formation
- Photons deviated through the gravitational lensing effect







An historical introduction to gravitational lensing

Sir Arthur Eddington





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An historical introduction to gravitational lensing



0-th order calculation



Lensing on CMB maps

$T(\hat{n}) \ (\pm 350 \mu K)$



$\mathbf{B}(\hat{n}) \ (\pm 2.5 \mu K)$

(no primordial B-modes)

credits D. Hanson

Unlensed

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Lensing on CMB maps

$T(\hat{n}) \ (\pm 350 \mu K)$



$E(\hat{n}) \ (\pm 25 \mu K)$

$\mathbf{B}(\hat{n}) \ (\pm 2.5 \mu K)$

(no primordial B-modes)

credits D. Hanson

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Exagerating a bit...



Lensing more in detail

• Photon deviation given by the gradient of a scalar field called lensing potential

$$\psi(\hat{\mathbf{n}}) \equiv -2 \int_0^{\chi_*} \mathrm{d}\chi \, \frac{f_K(\chi_* - \chi)}{f_K(\chi_*) f_K(\chi)} \Psi(\chi \hat{\mathbf{n}}; \eta_0 - \chi)$$



Lensing potential cosmological sensitivity

- Sensitive to parameters affecting structure formation rate
- Early dark energy, neutrino masses, modified gravity....
- One of the most important cosmological observable in the next years!



Effects on power spectrum

- Variance is preserved but power is reshuffled
- Small effect on Temperature, more important on E-modes
- B-modes: main source of signal, contaminant for inflationary signal detection
- Lensing correlates angular scales: non-Gaussian feature
- We can reconstruct the lensing potential from CMB





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1000

100



10

Effects on maps



Their difference shows a pattern similar to the LSS which lensed the signal



Effects on maps



Their difference shows a pattern similar to the LSS which lensed the signal



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Effects on maps



Their difference shows a pattern similar to the LSS which lensed the signal



• Lensing in real domain:

 $\Theta(\hat{\boldsymbol{n}}) = \tilde{\Theta}(\hat{\boldsymbol{n}} + \boldsymbol{d}(\hat{\boldsymbol{n}})) ,$ $(\boldsymbol{Q} \pm i\boldsymbol{U})(\hat{\boldsymbol{n}}) = (\boldsymbol{Q} \pm i\boldsymbol{\tilde{U}})[\hat{\boldsymbol{n}} + \boldsymbol{d}(\hat{\boldsymbol{n}})]$

- $d = \nabla \phi$
- Lensing is a convolution in harmonic domain:

• 6 different unbiased estimators:

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 $\langle d_{\alpha}(\boldsymbol{L}) \rangle_{\text{CMB}} = d(\boldsymbol{L}) \equiv L\phi(\boldsymbol{L})$

$$\langle d_{\alpha}(\mathbf{L}) d_{\beta}^{*}(\mathbf{L}') \rangle = (2\pi)^{2} \delta(\mathbf{L} - \mathbf{L}') (C_{L}^{dd} + N_{\alpha\beta}^{(0)}(L)$$

+ higher-order terms).

$$N_{\alpha\beta}(L) = L^{-2}A_{\alpha}(L)A_{\beta}(L) \int \frac{d^{2}l_{1}}{(2\pi)^{2}}F_{\alpha}(l_{1}, l_{2}) \Big(F_{\beta}(l_{1}, l_{2}) \\ \times C_{l_{1}}^{x_{\alpha}x_{\beta}}C_{l_{2}}^{x_{\alpha}'x_{\beta}'} + F_{\beta}(l_{2}, l_{1})C_{l_{1}}^{x_{\alpha}x_{\beta}'}C_{l_{2}}^{x_{\alpha}'x_{\beta}}\Big).$$

Breaking degeneracies with CMB lensing

• Lensing sensitive to geometry and late-time growth of structure: curvature



• Neutrino masses:



CMB lensing and large scale structures

• Planck established CMB lensing as a high-precision tool for cosmology



CMB lensing and large scale structures

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L'osservatorio Simons: 100 volte meglio

3-31

L'osservatorio Simons: 100 volte meglio



L'osservatorio Simons: 100 volte meglio



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Cosmological constraints of CMB polarization

 Dependency on cosmological parameters is similar but not the same: break parameter degeneracies!



Vector perturbations

- Vortical motion of the matter, i.e. with curl like velocity field
- Decay with I/a² while super horizon but can still affect CMB through vorticity of baryons or neutrinos



• Can produce B-modes very efficiently since they couple to dipole

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Polarization from vector perturbations

- Highly model dependent
- Various global topological defects



- Inhomogenous magnetic fields (to be motivated theoretically)
- Multi-field or brane inflation, second order perturbation theory


Cosmic Microwave Background: data analysis and experimental perspectives

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Petnica Cosmology Summer school 2017

Outline

- Some background on experimental CMB concepts
 - Data analysis chain
 - Instrumental systematics
 - Perspectives on recent experimental results
- Few more words on cosmological parameters
 - Maximum likelihood methods
 - Markov Chain Monte Carlo methods



Where do the data come from?



Ground

- Heavy hardware
 - large telescopes (resolution)
 - large receivers (sensitivity)
- Environment contamination (atmosphere, ground...)
- Cutting-edge technology
- Maintenance possible
- <IOy to deploy</pre>
- ~ I0 M€





Space

- Light hardware
- Extremely reliable technology
- Stable environment
- ~20y to deploy
- around G€
- full sky
- ~Only galactic contamination

Balloon are midway: (notably, limited atmosphere)

Current ongoing experiments I

Ground Based

	Chile	Have data	Current or planned freqs
*	ABS		145 GHz
	ACTPol/AdvACt		30, 40, 90, 150, 230 GHz
	POLARBEAR		90, 150 GHz
*	CLASS		40, 90, 150 GHz
	Antarctica		
*	BICEP/KECK		90, 150, 220 GHz
	SPTPol		90, 150 GHz
	QUBIC-Bolo int.	2016	90, 150, 220 GHz
	Elsewhere (for now)		
	B-Machine – WMRS		40 GHz
*	GroundBIRD, LiteBIRD	2016	150 GHz
*	GLP – Greenland	TBD	150, 210, 270 GHz
*	MuSE-Multimoded	TBD	44, 95, 145, 225, 275 GHz
	QUIJOTE – Canaries, HEN	1	11-20, 30 GHz

Current ongoing experiments II

Balloons



* EBEX LPSE

* PIPER

* SPIDER

Have data	Current or planned freqs		
	150, 250, 210 GHz		
TBD	5 chan 40-250 GHz		
2015	200, 270, 350, 600 GHz		
	90, 150, 280 GHz		





Pictures courtesy of Zigmund Kermish

DASI: first detection of CMB polarization





Map is 5 degrees square





Observation from the ground limited by atmospheric transparency



• Reduce as much as possible atmospheric emissivity: altitude and dryness



Data analysis overview



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• Time-Ordered data:

Polarbear: Volume = sampling rate x detector number x observation time = $\sim 100 \text{ Hz} \times \sim 1000 \text{ x} \sim 10^7 \text{ s} = \sim 10^{12} \text{ samples} (\sim 10 \text{ TB})$

- Map-making Planck HF maps: 1.7 arcmin resolution, full sky: 5 x 10⁷ pixels Polarbear: 1.7 arcmin, 0.1% sky: 5 x 10⁴ pixels
- Component separation
 Typically, information compression of O(1)
- Power spectrum estimation
 Typically O(10)-O(100) power spectrum points
- Estimation of O(1)-O(10) cosmological parameters

Compression has to be efficient and effective

 computer science and statistics play important roles

The POLARBEAR experiment

- CMB polarization dedicated experiment in Atacama Desert
- Targeting both large and small scales
- 80% of the sky with el > 30 accessible
- First season: deep 5x5 patches integration for lensing B-modes signal





Instrument design: POLARBEAR example



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Instrument characterization

• Estimate telescope pointing and optical response (antenna beam)



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Systematics error example

- Differential pointing: two detectors looking in different direction
- $\begin{aligned} d^{t}(t) &= g_{top} \left[I(\hat{n}(t)) + Q(\hat{n}(t)) \cos(2\psi(t)) + U(\hat{n}(t)) \sin(2\psi(t)) \right] \\ d^{b}(t) &= g_{bot} \left[I(\hat{n}(t)) Q(\hat{n}(t)) \cos(2\psi(t)) U(\hat{n}(t)) \sin(2\psi(t)) \right] \end{aligned}$
- Temperature to polarization leakage can prevent detection of B-modes
- Polarization modulation can reduce it







Systematics error example

- Differential pointing: two detectors looking in different direction
- $d^{t}(t)$ $= g_{top} \left[I(\hat{\mathbf{n}}(t)) + Q(\hat{\mathbf{n}}(t)) \cos(2\psi(t)) + U(\hat{\mathbf{n}}(t)) \sin(2\psi(t)) \right]$ $d^{b}(t) = g_{bot} \left[I(\hat{\mathbf{n}}(t)) - Q(\hat{\mathbf{n}}(t)) \cos(2\psi(t)) - U(\hat{\mathbf{n}}(t)) \sin(2\psi(t)) \right]$

(0,0)

-0.844

- Temperature to polarization leakage can prevent detection of B-modes
- Polarization modulation can reduce it

Difference of the 2 maps (1/4 day)

(0,0)

-0.844

μK



Systematic error examples



Systematic error examples



$\mathbf{d} = \mathbf{A} \cdot \mathbf{s} + \mathbf{n}$



$\mathbf{N} \equiv \langle \mathbf{n}^{\mathsf{T}} \mathbf{n} \rangle \longrightarrow \hat{\mathbf{s}} = (\mathbf{A}^{\mathsf{T}} \mathbf{N}^{-1} \mathbf{A})^{-1} \mathbf{A}^{\mathsf{T}} \mathbf{N}^{-1} \mathbf{d}$

- Numerically challenging, need to use advanced numerical linear algebra techniques
- Atmosphere = more problems

Courtesy of Julian Borril

$\mathbf{d} = \mathbf{A} \cdot \mathbf{s} + \mathbf{n}$

esa



$$\mathbf{N} \equiv \left\langle \mathbf{n}^{\mathsf{T}} \mathbf{n} \right\rangle \longrightarrow \hat{\mathbf{s}} = (\mathbf{A})^{\mathsf{T}} \mathbf{n}$$

- Numerically challenging, need to use advanced numerical linear algebra techniques
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esa

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EPOCH

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1E+09

1E+08

1985

COBE

1995

1990

• Atmosphere = more problems $\hat{s} = (A^T N^{-1} A)^{-1} A^T N^{-1} F d$ $(A^T N^{-1} F A) \hat{s}_{i^{th}} = A^T N^{-1} F d$



CMB fundamentals - PSI Cos

2020

2025

2030

2035

2015

BOOMERanG

2005

EPOCH

2010

2000

How to invert this $N_p \times N_p$ matrix? Inversion requires $N_p^3 \sim 10^{18}$ operations (100 cpu y)

Find approximate solution without explicit inversion using the Preconditioned Conjugate Gradient technique

Solve $\mathbf{B} \mathbf{x} = \mathbf{b}$ with \mathbf{B} symmetric positive definite.

Idea:

• use **B** as scalar product, given a search direction $\hat{\mathbf{p}}$ (with $\hat{\mathbf{p}}^t \mathbf{B} \hat{\mathbf{p}} = 1$), the projection onto it $\hat{\mathbf{p}}(\hat{\mathbf{p}}^t \mathbf{B} \mathbf{x}) = \hat{\mathbf{p}}(\hat{\mathbf{p}}^t \mathbf{b})$

can be interpreted as an approximate solution.

• project the solution on an increasingly larger subspace until the approximate solution is "good enough": e.g., $|\mathbf{B} \mathbf{x} - \mathbf{b}| / |\mathbf{b}| < 10^{-6}$







10⁰

10⁻¹

Flux (MJy/sr) 10-2

10-3

Free-free

10¹

^{10²} Frequency (GHz) 10³



10⁰

10⁻¹

Flux (MJy/sr) 10-2

10-3



 10^{0}

10⁻¹

Flux (MJy/sr) 10-2

10-3

Free-free

10¹

^{10²} Frequency (GHz) 10³







 10^{0}

10⁻¹

Flux (MJy/sr) 10-2

10-3

Free-free

10¹

^{10²} Frequency (GHz) 10³





 10^{0}



10⁰

10⁻¹

Flux (MJy/sr) 10-2

10-3





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Sv

_____ 10³

Free-free

10¹

^{10²} Frequency (GHz)

10-3

Component separation principle



Fig. 15- Eigi 25- Experimentation applitized application applitized applitized applitized applitized applitized applitized applitized applitized applitized application applitized applitized applitized applitized application application



Fig. 16. Bilight iles Beighterens temperature unes ion of the thorn of the diversity and ophysical contraction of the diversity of the diversi

Snapshot of foregrounds



Fig. 14. Maximum posterior intensity maps derived Planck collaboration X 2015). From left to right, top to bottom: CMB, synchrotron, free-free, spinning dust, thermal dust, line Gulio Fabbian CMB fundamentals - PSI Cosmology 2017

Snapshot of pola



Planck collaboration 2015



 μK_{CMB}

K_{RJ} @ 408 MHz

μK_{RJ} @ 30 GHz

250 Synchrotron

50

30

Synchrotron polarization

-250

10

0





cm-6 pc



0 K_{RJ}km⁻¹s



μK_{RJ} @ 545 GHz 20 0

10



Thermal dust



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1- Component separation generalities

(1)
$$map(\nu) = a_{cmb}(\nu)s_{cmb} + a_{dust}(\nu)s_{dust} + n(\nu)$$

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 $s \equiv \begin{bmatrix} Q \\ U \end{bmatrix}$

Component separation: a concrete example

1- Component separation generalities

(1)
$$map(\nu) = a_{cmb}(\nu)s_{cmb} + a_{dust}(\nu)s_{dust} + n(\nu)$$

for three frequency channels,
using matrix form
$$s \equiv \begin{bmatrix} Q \\ U \end{bmatrix}$$

(2)
$$\begin{bmatrix} map(\nu_1) \\ map(\nu_2) \\ map(\nu_3) \end{bmatrix} = \mathbf{A}(\nu_1, \nu_2, \nu_3) \begin{bmatrix} s_{cmb} \\ s_{dust} \end{bmatrix} + \begin{bmatrix} n(\nu_1) \\ n(\nu_2) \\ n(\nu_3) \end{bmatrix}$$

Component separation: a concrete example

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it's like map-making!

1- Component separation generalities

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it's like map-makingly
(3) $\begin{bmatrix} s_{cmb} \\ s_{dust} \end{bmatrix} = (\mathbf{A}^T \mathbf{N}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{N}^{-1} \begin{bmatrix} map(\nu_1) \\ map(\nu_2) \\ map(\nu_3) \end{bmatrix}$

1- Component separation generalities

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(2) $\begin{bmatrix} map(\nu_1) \\ map(\nu_2) \\ map(\nu_3) \end{bmatrix} = \mathbf{A}(\nu_1, \nu_2, \nu_3) \begin{bmatrix} s_{cmb} \\ s_{dust} \end{bmatrix} + \begin{bmatrix} n(\nu_1) \\ n(\nu_2) \\ n(\nu_3) \end{bmatrix}$
(2) $\begin{bmatrix} component separation is performed in two steps: 1- Estimation of the mixing matrix \mathbf{A} — method specific 2- Solving the linear system (2), with \mathbf{A} estimated above — general to any method
(3) $\begin{bmatrix} s_{cmb} \\ s_{dust} \end{bmatrix} = (\mathbf{A}^T \mathbf{N}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{N}^{-1} \begin{bmatrix} map(\nu_1) \\ map(\nu_2) \\ map(\nu_3) \end{bmatrix}$$

Planck foregrounds results

First polarized survey at high-frequency: highly polarized dust far from the galactic center



Planck foregrounds results

First polarized survey at high-frequency: highly polarized dust far from the galactic center



Power spectrum estimation in real life

Problem with $\hat{C}_{\ell} = \sum_{m=-\ell}^{\ell} \frac{a_{\ell m}^* a_{\ell m}}{2\ell+1}$: a_{lm} are not available (partial sky coverage)

 C_l is estimated by

- sampling the power of s along functions in the l subspace (the $\{Y_{lm}\}_{m \in \{-l,..,l\}}$ functions)
- averaging (the expected power on each function is C_l)

In this case the functions are orthogonal but don't have to be

If a function f straddles multiple l subspaces, the expected power of s along f is a (known) linear combination of C_l

Define a pseudo-basis of such f functions: $\tilde{Y}_{\ell m}$ (any set!) and sample s along them. $\tilde{a}_{\ell m} \equiv \int \tilde{Y}_{\ell m}^* s d\Omega$

$$\tilde{C}_{\ell} = \sum_{m=-\ell}^{\ell} \frac{\tilde{a}_{\ell m}^* \tilde{a}_{\ell m}}{2\ell + 1}$$
 Pseudo-power spectrum

$$\langle \tilde{C}_{\ell} \rangle = \sum_{\ell'} M_{\ell\ell'} C_{\ell'}$$

Compute and "invert" the M_{ll} and and you are done

Hauser and Peebles (1973), Hivon et al. (2002), Kogut et al. (2003)

Notable example: $\tilde{Y}_{\ell m} \equiv WY_{\ell m}$ W is the inverse noise

- handle cut sky
- handle inhomogeneous coverage
- Computing the M_{ll} scales as $\mathcal{N}_p^{3/2}$

Power spectrum estimates errors



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Data analysis basics

- Scientific observations can be represented by a data vector d which can be a time series {t} or temperature map of the sky {T} or something else.
- Data have information about some physical process for which we have a theoretical model represented by a set of parameters i.e., parameter vector Θ.
- One of its example is a Gaussian process represented by two parameters i.e., the mean μ and the variance σ². The probability of obtaining data d given a theoretical (Gaussian model (μ, σ²)) is given by:

$$P(d|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\frac{(d-\mu)^2}{\sigma^2}\right]$$

Data analysis basics - II

- Not all the inverse problems are solvable and in some cases we can easily find out why that is so.
- On the basis of whether the size N of the data vector d is larger, smaller or equal to the size M of the parameter vector
 Θ, there are three possibilities.
 - N = M : Unique solution is possible
 - N > M: Over constrained problem, χ^2 minimization, Unique solution
 - N < M : Under-constrained problem, ill posed problem, priors, regularization

Note that in the above consideration we have assumed that all the data points are independent.

One of the common methods to solve an inverse is to minimize a measure of misfit between the data and the theoretical model.

Maximum likelihood estimation

The probability distribution P(Θ|d) (posterior) for model parameters Θ given data d can be related to the probability P(d|Θ) (likelihood) of an experiment giving data d for model parameters Θ using the Bayes' theorem:

$$P(\Theta|\mathbf{d}) = rac{P(\mathbf{d}|\Theta)P(\Theta)}{P(\mathbf{d})}$$

where $P(\Theta)$ is called the prior and $P(\mathbf{d}) = \sum P(\mathbf{d}|\Theta)P(\Theta)$ is used for the normalization purpose.

For the case of flat prior, posterior and likelihood are proportional:

$$P(\Theta|\mathbf{d}) = P(\mathbf{d}|\Theta) = L(\mathbf{d}|\Theta)$$

In Bayesian formalism we can easily incorporate new data in analysis by considering the posterior of the old data as prior.

The role of the prior



Note that when likelihood/posterior is not Gaussian then the average value of the parameter $< \theta >$ may not coincide with the value of θ_0 at which the likelihood/posterior is maximum.

An efficient sampling: Markov Chains MC

Once we have the probability distribution P(Θ|d) for model parameters Θ we can statistics of the parameters:

$$<\Theta>=\int\Theta d\Theta P(\Theta|\mathbf{d})$$

In practice, before carrying out the above integral we find out the one dimensional probability distribution by marginalization over other parameters:

$$P(\theta_r) = \int d\theta_1 d\theta_2 \dots d\theta_{r-1} d\theta_{r+1} \dots d\theta_M P(\theta_1, \theta_2, \dots, s\theta_M)$$

and

$$< heta_r>=\int heta_r d heta_r P(heta_r)$$

Maximum likelihood estimation

- Carrying out multi-dimensional integration is very expansive i.e., computational cost grows as O(n^M) where n is the number of grid points along one direction and M is the dimensionality of the parameter space.
- If we can replace the multi-dimensional integration by summation over a finite number of points which represent the probability distribution function then computational cost becomes manageable.

$$<\Theta>=\int\Theta d\Theta P(\Theta|\mathbf{d})=rac{1}{N}\sum_{i=1}^{N}\Theta_{i}P(\Theta_{i}|\mathbf{d})$$

Markov-Chain Monte Carlo sampling samples the likelihood function in such a way that there are more point in the region where the likelihood function has the large values and less where it has small values.

Basic idea of MCMC

- When we toss a coin n times then the outcome of the nth toss does not depend on the outcome of any of the previous outcomes.
- In a Markov-Chain the probability of a random variable X_n to have value x_n at step n depends on the probability of the variable X_{n-1} to have the value x_{n-1} at step n-1.

$$P(X_N) = P(X_n, X_{n-1})P(X_{n-1})$$

where $P(X_n, X_{n-1})$ is called the transition probability, transition kernel or proposal density.

In most cases transition kernel is symmetric:

$$P(X_n, X_{n-1}) = P(X_{n-1}, X_n)$$

The transition probability $P(X_n, X_{n-1})$ has the remarkable property that after an initial burn-in period it generates a sample which has the probability distribution P(X).

CMB gaussian likelihood

In terms of C_l the likelihood function can be written as:

$$L(T|C_{l}) = \prod_{lm} \frac{1}{\sqrt{C_{l}}} \exp[-|a_{lm}|^{2}/(2C_{l})]$$

Since we observe only one sky so we cannot measure the power spectra directly, but instead form the rotationally invariant estimators, C_I, for full-sky CMB maps given by

$$\hat{C}_{l} = \frac{1}{2l+1} \sum_{m=-l}^{m=l} |a_{lm}|^{2}$$

$$\chi^2 = -2\log L(\hat{C}_l|C_l) = \sum_l (2l+1) \left[\log \left(\frac{C_l}{\hat{C}_l}\right) + \frac{\hat{C}_l}{C_l} - 1 \right]$$

MCMC chain convergence





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Current strategies to detect B-modes

• 2-point correlation: CMB power spectrum

3-point correlation: CMB cross correlation with biased tracers of dark matter halos

4-point correlation: lensing reconstruction with polarization











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Ground-based sensitivity: POLARBEAR example



POLARBEAR collaboration 2014

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SPTpol: B-modes through cross-correlation

- Cosmic infrared background and lensing potential are correlated
- Idea: construct a noisy template of lensing B-modes and correlate with my observed B-modes hidden within noise



Lensing reconstruction with polarization data

• Clean reconstruction of the deflection field from CMB polarization

$$d_{EB}(\mathbf{L}) = \frac{A_{EB}(L)}{L} \int \frac{d^2 \mathbf{l}}{(2\pi)^2} E(\mathbf{l}) B(\mathbf{l}') \frac{\tilde{C}_l^{EE} \mathbf{L} \cdot \mathbf{l}}{C_l^{EE} C_{l'}^{BB}} \sin 2\phi_{\mathbf{l}\mathbf{l}'}$$
$$= \nabla \phi$$
$$d_{EE}(\mathbf{L}) = \frac{A_{EE}(L)}{L} \int \frac{d^2 \mathbf{l}}{(2\pi)^2} E(\mathbf{l}) E(\mathbf{l}') \frac{\tilde{C}_l^{EE} \mathbf{L} \cdot \mathbf{l}}{C_l^{EE} C_{l'}^{EE}} \cos 2\phi_{\mathbf{l}\mathbf{l}'}$$

Detection of lensing of CMB polarization from CMB alone



d

POLARBEAR direct B-modes measurement

• First evidence for B-modes power spectrum (ApJ 2014)

 $A_{BB} = 1.12 \pm 0.61 (\text{stat})^{+0.04}_{-0.10} (\text{sys}) \pm 0.07 (\text{multi})$

Polarization angle self-calibration from detected EB power (~I deg)



Cosmological constraints with B-modes

- Constraints on inflationary magnetogenesis: compatible with lower r and blue tensor spectra
- Cosmic defects / vector plus tensor modes can explain[±]
 data but rule out local strings
- Alternatives to inflation can be tightly constrained:
 - string gas cosmology with blue tensor spectrum
 - slow roll or null energy condition violation etc....





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ACTpol results I (148 GHz)



ACTpol results II (148GHz)

- High resolution allows constraining diffuse polarized emission from astrophysical sources
- No B-modes constraints reported yet





Benson et al 2014

- Highest resolution among current experiment (larcmin)
- Cluster science together with CMB science
- No direct B-modes measurement reported



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 Highest resolution among current experiment (larcmin)

- Cluster science together with CMB science
- No direct B-modes measurement reported



Benson et al 2014



 Highest resolution among current experiment (larcmin)

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Benson et al 2014

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SPTpol

150 GHz

10 deg²

SPT-CL J2341-5724 (z = 1.36)

SPT-CL J2329-5831 ((z = 0.81)

SPTpol maps 2017



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SPTpol lensing results

Lensing reconstruction with polarization (still dominated by temperature sensitivity)


ACTpol lensing results

- Lensing reconstruction using temperature and polarization
- Cross-correlation with Planck CIB at small angular scales, additional constraints for CIB model at small angular scales
- Indirect constraints of lensing B-modes consistent with POLARBEAR and SPTpol results



BICEP2/Keck

- Dedicated to large s of subsequent upgra
- Reported evidence $(-0.01 \Big|_{BB jack \chi^2 PTE = 0.96} \\ 0 100 200 \\ interpreted by prime una v v v$





B-modes delensing



(J2000)

Dec

The future of ground-based CMB experiment

- Multichroic pixels receivers: 7,588 detectors, 95/150 GHz e.g. for POLARBEAR-2
- Simons Array by 2018: 3 telescopes, 22,764 detectors, 95/150/220 GHz and high resolution (similar, AdvACTpol, SPT3G)
- High sensitivity B-modes characterization on all angular scales
- 500.000 detectors telescope network CMB-S4







Towards "Stage 4" experiment

- Large sky, overlap with other astrophysical surveys for cross-correlation studies
- Constrain neutrino mass hierarchy, galaxy mass bias, dark energy....
- I% error constraints on r with internal delensing capabilities
- Cosmic birefringence, primordial magnetic fields...



Conclusions

- CMB is one of the most accurate method available to constrain cosmology (both primordial and intermediate epochs)
- CMB field IS NOT dead: the B-modes era will be dominated by suborbital experiments for the next years to come
- Cross correlation: among the most active area of research nowadays (both LSS and primordial science)
- Early universe constraints will face dramatic improvement in few years
- Foreground (galactic and extragalactic) crucial for experimental success



Towards Simons Observatory and CMB-S4

Some material courtesy of Silvia Galli, Davide Poletti, Jayanti Prasad, Josquin Errard

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Towards Simons Observatory and CMB-S4



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