

Dark Matter - Problem set

Petnica Summer Institute - 2018

Problem I

One of the first evidences for the existence of Dark Matter came from the rotation curves of spiral galaxies. What kind of circular velocity profile, $V_{\text{cir}}(R)$, do you expect to for the following matter density distributions:

1. Sphere of constant density: $\rho(r) = \rho_0 \cdot \theta(R_{\text{max}} - r)$
2. Axisymmetric razor-thin disk: $\rho(R) = \Sigma_0 \cdot e^{R/R_h}$

What kind of spherically symmetric density distribution would lead to constant circular velocity profile $V_{\text{cir}}(R) = V_0$?

Problem II

Warm Dark Matter can not form structure below the free streaming length which is defined as follows:

$$\lambda_{\text{FS}} = \int_0^{t_{\text{EQ}}} \frac{v(t)}{a(t)} dt \quad (1)$$

After the particles become non-relativistic their velocity redshifts as $v(t) = \frac{c \cdot a(t_{\text{NR}})}{a(t)}$. Compute the free streaming length of Dark Matter that decouples at $a_{\text{NR}}(t) = \frac{3k_{\text{B}}T_0}{mc^2}$ for $m = 1 \text{ eV}$ and $m = 1 \text{ MeV}$. The scale factor at radiation-matter equality is approximately $a(t_{\text{EQ}}) \approx \frac{1}{3500}$.

Problem III

Pauli exclusion principle prevents fermions to collapse in an arbitrarily dense configuration. Assuming that the Dark Matter is in the form of degenerate Fermi gas

$$f(p) = \begin{cases} 1; & p \leq p_F \\ 0; & p > p_F \end{cases}, \quad (2)$$

estimate the minimum particle mass that is still compatible with halo of mass M and radius R . You can approximate the density of degenerate Fermi gas throughout the halo is constant.

Problem IV

Weakly Interacting Massive Particles (WIMPs) generically yield the correct abundance of Dark Matter for a typically velocity averaged weak cross section $\langle \sigma v \rangle \sim G_{\text{F}}^2 m^2$. By using the expression for number density and energy density of particles in thermal equilibrium:

$$n_{\text{eq}} = \int_0^\infty d^3p f(p) \quad (3)$$

$$\rho_{\text{eq}} = \int_0^\infty d^3p E(p) f(p) \quad (4)$$

compute the freeze-out temperature of WIMPs, which were initially in equilibrium with the plasma and decoupled during the radiation dominated era. What is their energy density today?¹

¹Hint: WIMPs decouple in the radiation dominated era. Their annihilation rate is given by $\Gamma = \langle \sigma v \rangle n_x$. After the decoupling their number density scales as $n_x \propto T^{-3}$.