Exercise on Cosmology

(Dated: August 5, 2024)

Constrain cosmological parameters from SNIa and BAO "data".

Package requirements

I suggest installing `python` via https://docs.anaconda.com/free/anaconda/install/index.html. I also suggest creating an environment specific to this exercise, or in general this Summer School, so that you will:

- learn to use the good practice of environments;
- any stuff you install for this won't "pollute" your general conda installation (see point above).

A very useful link is of course the cheat sheet, and also this. The packages that will be needed are `numpy` (of course), `matplotlib` (for plots), `scipy` (for integrals, function minimization, etc.). For the sampling and derivation of constraints I suggest:

- `emcee`. See e.g. this link, which is essentially all you need to carry out this exercise;
- `getdist`. See this and this. Once you have the MCMC chains, this has more fancy methods of plot and analysis.

Both can be installed via conda with

```
conda install -c conda-forge emcee
conda install -c conda-forge getdist
```

Always install them in your specific environment! Just run the above commands once you have gone to your environment via "`conda activate my_env`". It is a good practice!

Physics (luminosity distance and energy budget)

We need Standard Candles to measure the distance. Let's recall that the luminosity distance is defined such as

$$F = \frac{L}{4\pi \underbrace{(1+z)^2 r^2}_{d_L^2}} \, , \tag{1}$$

where F is the observed flux, L is the object's luminosity, and

- in a flat universe, $4\pi r^2$ is the proper area of a sphere that is drawn around the object and crosses the Earth at observation time t_0. I.e. as if the object and the observer were both at t_0;
- we get a factor of $1/(1+z)$ because of the difference in the energy of photons at the reception point from the energy at emission;
- the same factor enters in the expression for the number of photons crossing a unit receiving area in a unit time, since the time intervals for the source and observer differ by a factor $1/(1+z)$.

Recall that in conformal time

$$\mathrm{d}s^2 = a^2(\eta)\Big(-\mathrm{d}\eta^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\Big) \, . \tag{2}$$

For a curved universe, replace r^2 by

$$r^2 \to \begin{cases} \frac{\sin^2(\sqrt{K}\, r)}{K} & \text{for } K > 0 \, , \\ \frac{\sinh^2(\sqrt{|K|}\, r)}{|K|} & \text{for } K < 0 \, . \end{cases} \tag{3}$$

Then, we need to compute r^2. Photons follow lightlike paths $\mathrm{d}s^2 = 0$ so, fixing $\mathrm{d}\Omega = 0$ since they have a fixed direction, the FLRW metric above tells us that (reinstating units, c is the speed of light $c \approx 3 \times 10^5\,\mathrm{km/s}$)

$$r = c\int_0^z \mathrm{d}z' \frac{1}{H(z')} \ , \tag{4}$$

where the Friedmann equations (neglecting radiation) tell us that

$$H(z) = H_0\sqrt{(\Omega_\Lambda)_0 + (\Omega_K)_0(1+z)^2 + (\Omega_\mathrm{m})_0(1+z)^3} \ , \tag{5}$$

where $(\Omega_\Lambda)_0 + (\Omega_K)_0 + (\Omega_\mathrm{m})_0 = 1$! We also need K if we are in a curved universe: we recall that at a generic redshift the fractional contribution of spatial curvature to the energy density is

$$\Omega_K = -\frac{K}{(aH)^2} \ , \tag{6}$$

hence

$$K = -(\Omega_K)_0 H_0^2 \ . \tag{7}$$

From now on we drop the subscript "0" on Ωs.

In the exercise we will focus on a flat universe, with "dark energy" following the "w_0-w_a" parameterization. See e.g. https://arxiv.org/abs/1411.1074v3. For this universe we have

$$H(z) = H_0\sqrt{(1-\Omega_\mathrm{m})(1+z)^{3(1+w_0+w_a)}\mathrm{e}^{-\frac{3w_a z}{1+z}} + \Omega_\mathrm{m}(1+z)^3} \ . \tag{8}$$

I don't really understand the physics of this parameterization, ask Marko for more details on it. We will focus on $w_a = 0$, and the goal is measuring Ω_m and w_0, and discover if the "Universe" I made up has a cosmological constant ($w_0 = -1$) or not.

Then, we observe fluxes and obtain distances if we know L of the object. Some examples are Cepheids and SNIa (see below, the section after BAO).

Physics (BAO)

As Marko explained, we have a feature in the matter power spectrum. Given a reference point (e.g. an overdensity), there is a preferential clustering of galaxies at a distance r_d from it. If we know ω_b (recall that $\omega_\mathrm{species} = \Omega_\mathrm{species}h^2$), we have

$$r_d = 55.154\frac{\mathrm{e}^{-72.3(\omega_\nu+0.0006)^2}}{(\Omega_m h^2)^{0.25351}\omega_b^{0.12807}}\,\mathrm{Mpc} \ ,$$

where

$$\omega_\nu = 0.0107 \times \frac{\sum_\nu m_\nu}{1.0\,\mathrm{eV}} \ .$$

Then, if we measure the angular size of this "standard ruler" of which we know the size via the formula above, we can infer distance to it using the angular diameter distance.

In practice, what we measure best is α,[1] and this is related to the "volume-averaged distance" by

$$\alpha = \frac{D_V}{r_d}\frac{r_{d,\mathrm{fid}}}{D_{V,\mathrm{fid}}} \ ,$$

where

$$D_V = \left[zD_H(z)D_M^2(z)\right]^{\frac{1}{3}} \ ,$$

[1] Essentially, the conversion between angles and distances is different depending on whether we observe the clustering between galaxies whose separation vector is orthogonal to the line of sight. The most well-measured "angle" is this α.

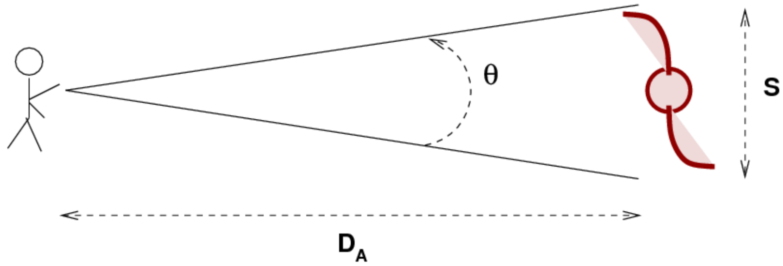

Figure 1: Angular diameter distance.

with

$$D_H(z) = \frac{c}{H(z)} \ ,$$

the comoving angular diameter distance in flat space (equal to the comoving distance in flat space) being

$$D_M(z) = c\int_0^z \mathrm{d}z' \frac{1}{H(z')} \ .$$

Here, "fid" denotes a fiducial cosmology that we use to convert from measured angles and redshifts to distances. We use the convention $h_{\rm fid} = 0.7$, $\Omega_{m,\rm fid} = 0.3$, $w_{0,\rm fid} = -1$ and $w_{a,\rm fid} = 0$. *These have nothing to do with the cosmological parameters you want to infer!* We fix also

$$\sum_\nu m_\nu = 0.06\,\mathrm{eV}$$

and (CMB prior: this quantity is measured *very* well from CMB temperature anisotropies)

$$\omega_b = 0.0224 \ .$$

Finally, recall that for a flat universe

$$H(z) = H_0\sqrt{(1-\Omega_{\rm m}) + \Omega_{\rm m}(1+z)^3} \ .$$

For a flat w_0-w_a universe, instead, we have (https://arxiv.org/abs/1411.1074v3)

$$H(z) = H_0\sqrt{(1-\Omega_{\rm m})(1+z)^{3(1+w_0+w_a)}\mathrm{e}^{-\frac{3w_a z}{1+z}} + \Omega_{\rm m}(1+z)^3} \ .$$

Standard candles

Cepheids

These have intrinsic L between $400L_\odot$ and $40000L_\odot$. So apparently not very standardized. But: looking at Cepheids in the Small Magellanic Cloud (which covers a small redshift, so we can forget about variations in d_L^2 when converting from flux to L), Leavitt found a relation between period of pulsation and mean brightness. Hence, if you measure period, you can infer brightness, if you assume all Cepheids behave the same. But, you still need to calibrate the period-luminosity relation! Otherwise you can only measure ratios of d_L^2 between Cepheids. This could be done if you have a close Cepheid, measure the flux, and measure the parallax distance d_π, and infer luminosity via $f = L/(4\pi d_\pi^2)$. It is tricky to use these stars if they are not in the Hubble flow, and peculiar velocities must be accounted for. This happens if $z \lesssim 0.02$. Let's turn to SNIa then!

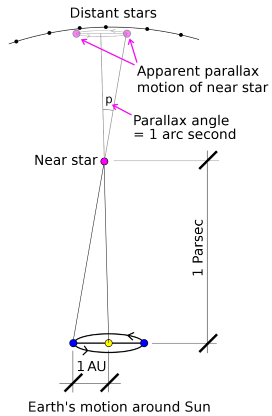

Figure 2: Stellar parallax.

SNIa

They have intrinsic L around $4 \times 10^9 L_\odot$. But "around" is bad. We would like that all SNIa have same luminosity! How do we determine it? The shape of the light curve (which should be how luminosity varies with time around the explosion) is correlated with the peak luminosity. If light curve shoots up and declines down rapidly, SNIa is less luminous, and vice versa. Also here you need to calibrate. You find a set of SNIa whose distance you find via observation of Cepheids in same galaxy, for example.

One more thing is apparent magnitude and absolute magnitude. Apparent magnitude m is defined as

$$m = -2.5 \log_{10}(f/f_x) \ , \tag{9}$$

where the reference flux is $f_x = 2.53 \times 10^{-8}\,\mathrm{watt\,m^{-2}}$. The absolute magnitude is defined as the apparent magnitude it would have at a luminosity distance of 10 pc, i.e.

$$M = -2.5 \log_{10}(L/L_x) \ , \tag{10}$$

where $L_x = 78.7 L_\odot$, so that an object that produces $f = f_x$ and is at $d_L = 10\,\mathrm{pc}$ would have $L = L_x$. Then, one can easily see that the difference between apparent and absolute magnitudes μ is

$$\mu = 5 \log_{10}\left(\frac{d_\mathrm{L}}{10\,\mathrm{pc}}\right) = 5 \log_{10}\left(\frac{d_\mathrm{L}}{1\,\mathrm{Mpc}}\right) + 25 \ . \tag{11}$$

If we measure fluxes and know luminosities, we can get μ, and from here get d_L. Anyway, in the exercise, since I provide made-up data, I give you directly the fluxes. If I were to give you data from e.g. http://supernova.lbl.gov/Union/figures/SCPUnion2_mu_vs_z.txt, you would get a list of redshifts (the error on the redshift must have already been accounted for), of difference between apparent and absolute magnitudes μ, and errors on these.

Exercise

The data include a list of redshifts, of fluxes, and the errors on fluxes. We assume that errors are uncorrelated between redshifts, and Gaussian. From these data, you can try to measure the quantities Ω_m and w_0. Recall that in a flat universe we have

$$d_\mathrm{L} = (1+z) d^\mathrm{co} \ , \tag{12}$$

where the comoving distance to redshift z is given by

$$d^\mathrm{co} = c \int_0^z \mathrm{d}z' \frac{1}{H(z')} \ , \tag{13}$$

where c is the speed of light $c \approx 3 \times 10^5$ km/s and

$$H(z) = H_0 \sqrt{(1-\Omega_\mathrm{m})(1+z)^{3(1+w_0+w_a)} \mathrm{e}^{-\frac{3 w_a z}{1+z}} + \Omega_\mathrm{m}(1+z)^3} \ , \tag{14}$$

where you can fix $w_a = 0$. Importantly, h is defined by

$$H_0 = h \times 100.0 \, \frac{\text{km/s}}{\text{Mpc}} \ . \tag{15}$$

You can fix $h = 0.67$ in the exercise.

From these formulas you are able to compute the luminosity distance in Mpc, which is what you need to analyze the data and infer $\Omega_{\rm m}$ and w_0. Again, you can assume zero spatial curvature, $h = 0.67$ and $w_a = 0$ for this exercise. You will see that with only SNIa data you cannot constrain these quantities, there is a lot of degeneracy between these two parameters. In other words, you cannot nail down if you have a cosmological constant or not.

You will see that the same happens for BAO, where recall that the thing you measure is α, defined as

$$\alpha = \frac{D_V}{r_d} \frac{r_{d,\rm fid}}{D_{V,\rm fid}} \ . \tag{16}$$

In the exercise I give you a set of redshifts (DESI-like), a set of corresponding measured αs, and errors on these measurements. You can try to infer also here $\Omega_{\rm m}$ and w_0 and see what happens. Also in this case you shouldn't be able to nail down this "Universe".

If you combine the datasets, instead. . .